

### ART. III – *Observations on the Geometry of Roman Camps*

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THE fortifying of strategically important sites was practised from remote antiquity, yet the idea of temporary field works seems to have been a relatively late development. Hannibal credited Pyrrhus, King of Epirus (319-272 BC), with perfecting the art and rated him therefore as second only to Alexander as a general.<sup>1</sup> Another tradition maintains that Pyrrhus learned it from the Romans,<sup>2</sup> so perhaps the truth is that each gained something from the other. In the Punic Wars, fortified camps were used by both sides but the earliest archaeological evidence of Roman field fortification comes from the parallelogram shaped camps at Renieblas, near Numantia in Spain, and dating from the mid-2nd century BC.<sup>3</sup>

Ancient armies, like all others, required careful management in the field, with organisational methods closely related to battle order and tactics. Drills were needed to enable complex tasks and manoeuvres to be performed quickly, and the advent of field fortification must have required new routines. Not only were the arts of trenching and rampart construction to be perfected but also those of site survey and measurement. These procedures could be somewhat ponderous and so had to be made both simple and rapid. At the same time, the associated drill had to be sufficiently flexible to meet a wide range of troop numbers and site conditions.

Apart from the tactical factors affecting the choice of site, the other key considerations would appear to be, first, the camp area – depended on size of force; second, the internal lay out – depended on its organisation; and third, the shape of the defended perimeter – depended on the ground. In short, what was needed was a method of knowing what area the camp should be, its shape and how it should be marked out internally, so that all knew their proper place. The method had also to operate by some simple rules and produce clear instructions to the men. The Roman army response to this challenge was an extraordinarily simple, yet durable, model which is still detectable in its innumerable camps and forts.

### **Roman Camps**

The general features of Roman camps have been summarised by Collingwood and Richmond.<sup>4</sup> They have straight sides, rounded corners and a generally rectangular outline; the reference by Vegetius<sup>5</sup> to curved ramparts probably reflects barbarian influences at work in the late imperial army. They vary greatly in area according to the size of the force encamped but are always quadrilateral in shape, if not actually symmetrically rectangular. Josephus states that setting out a rectangle was always the first step<sup>6</sup> and the camp described by Polybius<sup>7</sup> was square, while Hyginus<sup>8</sup> refers to those with sides, or axes, in the ratio of 2:3. Many camps of greatly varying area have now been described and a recent monograph from the Royal Commission on the Historical Monuments of England describes the authenticated camps in England.<sup>9</sup>

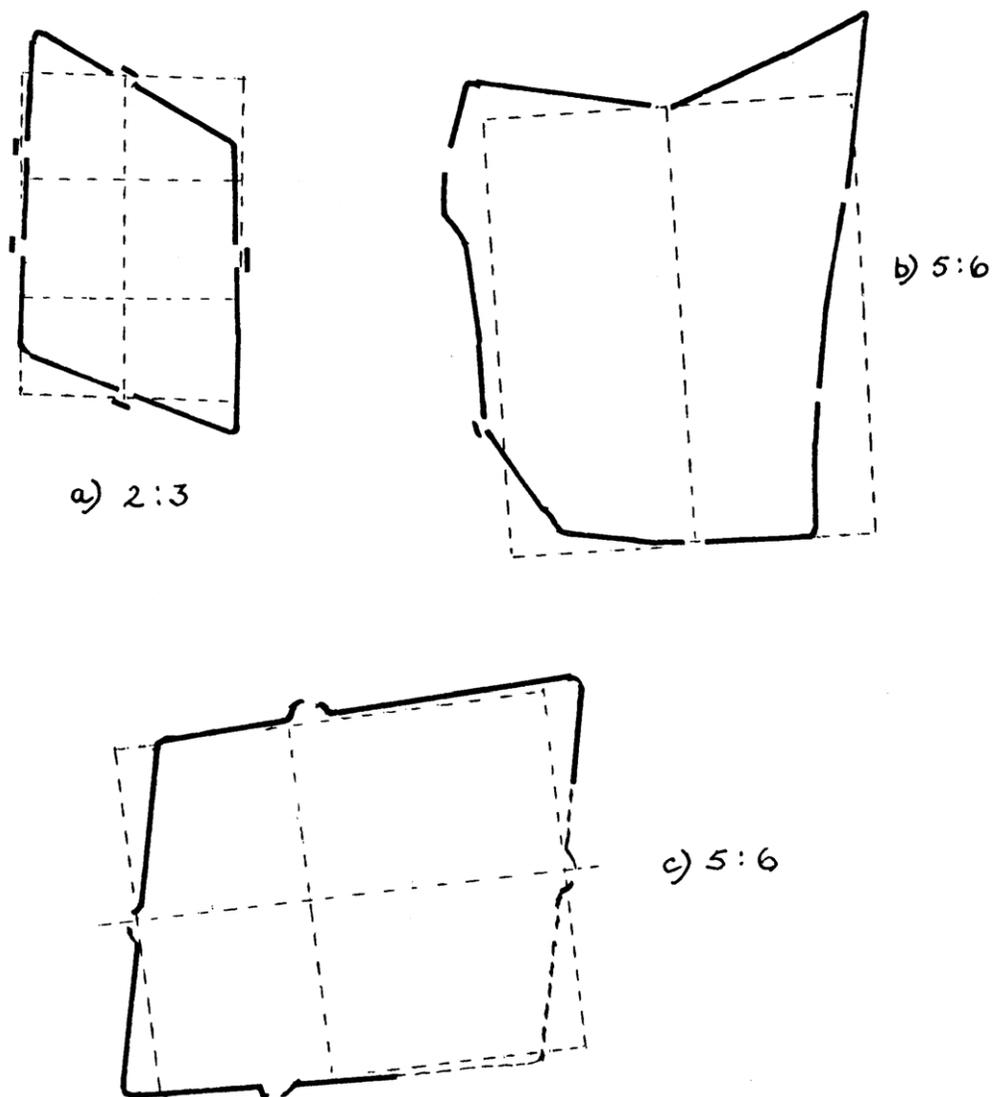


FIG. 1. (a) Featherwood West, (b) Rae Dykes, (c) Stracathro  
(Not to scale).

The published plans show that the rectangular plan was often modified by realignment of one or more of the ramparts to give parallelograms, trapezoids or asymmetrical outlines. The reasons for these modifications are not always obvious, especially with long vanished camps detected by aerial photography but it was almost certainly to make the best use of the ground, avoiding such obstacles as bogs and rock outcrops. Rey Cross, for example, shows extensive modifications which made best use of the flat land between the River Greta and the marshy moorland to the north<sup>10</sup>. Moreover, inspection of the plans reveals that the modifications are nothing more than a simple rotation of a rampart about a point on the perimeter of a symmetrical basic rectangle and they have an insignificant effect on the area. It seems that this simple device accounts for all the odd-shaped camps recorded in the literature. Some other examples are shown in Fig. 1.

It is clear, then, that the Romans met two of the criteria cited above, laying out camps of different area and modifying their outline to take account of the ground. Internal organisation was also methodically addressed, as is clear from the somewhat empirical accounts given by Polybius, Josephus, Hyginus and Vegetius. Once the site was chosen, a reference point before the *praetorium* was fixed and the *groma* set up. Two axes set at right angles were then marked out, the *decumanus* or longitudinal axis, and the transverse axis, or *cardo*. The perimeter and an internal matrix of lines, running parallel to the axes, at 60 Roman feet intervals were then laid out by officers known as *mensores*,<sup>11</sup> using 10 foot measuring rods and 6 foot spears, and checked by cross sightings using the *groma*. Within certain 60 foot strips, called *strigae*, the tents were pitched. Other strips formed the internal roads. Each unit knew its location within the grid, almost certainly by means of co-ordinates expressed in terms of distances from the axes, as in the case of farm holdings on centuriated land.<sup>12</sup>

The rectangle upon which a camp is based may be found in a number of ways. With regularly symmetrical camps it is simply a matter of observation. In camps with two parallel sides, the perpendicular distance between them gives one dimension, while averaging the lengths of the other two sides gives the second. With asymmetrical outlines (e.g. Rey Cross, Crackenthorpe and Plumpton Head) it is necessary to take the average of opposite ramparts. The base rectangle may then be drawn in (see Fig. 1) and its area determined simply by multiplying together the lengths of the long and short axes (or sides). If the dimensions are expressed in feet, yards or metres, the area values may become large and clumsy to handle and it is perhaps for this reason that the Romans, when dealing with farm land, expressed the linear dimension in *actus* of 120 Roman feet (1 Roman foot = 11.64 inches) and areas in square *actus*, or *actus quadrati*. According to Dilke,<sup>13</sup> there is no textual evidence of the *actus quadratus* in the military context, but even larger area units were applied to agricultural land (*heredia*), and the use of arithmetic shorthand to handle large numbers was also well established.<sup>14</sup>

The base rectangles of a number of camps derived by the method given above are set out in Table 1. It is important to note that the values refer to the *base rectangles* and not to the actual areas defined by the ramparts. The observed values for *cardo* (C) and *decumanus* (D) usually yield an area (A) which approximates to a whole number of *actus quadrati* and the fractional values are probably due to observational, or perhaps original surveying error. For this reason the table also shows the presumed area (in whole numbers), the rederived values for C and D and the

percentage errors, which are very small. These observations suggest that the base rectangle defines the rampart, probably its inner face rather than the inner margin of the *inter vallum* road.

TABLE 1  
Dimensions and Areas of Some Roman Camps  
Lengths in Roman feet and areas in *actus quadrati*

Camp & a. ratio	Observed values			Presumed	Rederived values (% error)			
	D	C	Area	Area	D	(e)	C	(e)
1:1								
Polybius <sup>15</sup>	2017	2017	282.52	280	2008	0.45	2008	0.45
Abernethy <sup>16</sup>	2028	2028	285	285	2028	0.0	2028	0.0
2:3								
Masada F1 <sup>17</sup>	505	337	11.82	12	509	0.77	339	0.77
Keithoch <sup>18</sup>	1577	105	115.2	115	1570	0.9	1051	0.9
Esgairp'd <sup>19</sup>	740	493	25.34	25	735	0.67	490	0.67
Ardoch 4 <sup>20</sup>	1831	1221	155.29	155	1826	0.09	1220	0.09
Ardoch 6 <sup>21</sup>	2463	1642	280	280	2463	0.0	1642	0.0
St Harmon <sup>22</sup>	926	617	39.69	40	930	0.38	617	0.38
Featherwood <sup>23</sup>	1606	1071	119	120	1610	0.23	1073	0.23
4:5								
Troutbeck <sup>24</sup>	710	568	28.01	28	710	0.02	568	0.01
Caerau <sup>25</sup>	1199	956	79.93	80	1200	0.04	960	0.04
Lofshaw Hill <sup>26</sup>	1522	1217	128.63	128	1518	0.24	1214	0.25
Llwyn-y-Brain <sup>27</sup>	614	491	20.97	20	600	2.4	480	2.4
Brampton Bryan <sup>28</sup>	1997	1598	221.67	220	1990	0.38	1592	0.38
Masada B <sup>29</sup>	575	449	17.93	18	569	1.02	455	1.4
5:6								
Longthorpe <sup>30</sup>	1569	1307	142.45	142	1566	0.16	1305	0.16
Rae Dykes <sup>31</sup>	2221	1851	285.4	285	2219	0.07	1849	0.07
Strachathro <sup>32</sup>	1594	1286	138.26	140	1555	2.4	1285	0.0

The relationship of the lengths of the sides (or axes) of a rectangle is termed the *aspect ratio*, and in this series the ratios were 1:1 (square), 2:3, 4:5 and 5:6, an arrangement detectable even in the far-from-standard outline of Rae Dykes. Furthermore, the area values are not widely variable but fall into definite categories which obviously reflect the size of the occupying forces. They may be arranged in series which, not surprisingly, indicate that specific formations regularly operated in the field. Table 2 lists five series in which the camp areas increase by a factor of two. See Welfare and Swan<sup>33</sup> for camp sizes expressed in acreages.

TABLE 2

## Camps whose Approximate Areas vary by a Factor of Two

(Acres in a.q. in brackets)

1. (12) Masada F1 – (24 or 25) Esgairperfedd.
2. (110) Keithock – (220) Brampton Bryan.
3. (c.70) Rey Cross, Crackenthorpe, Plumpton Head – (140) Longthorpe. Stracathro – (280) Polybius, Abernethy, Ardock 6, Rae Dykes.
4. (30) Troutbeck – (60) missing – (120) Featherwood.
5. (20) Llwyn-y-Brain – (40) St Harmon, – (80) Caerau, – (160) Ardoch 4.

It seems clear that camp areas were not merely guessed at, or even roughly estimated, but were calculated according to need. To have done otherwise, by resorting to guesswork, would have often resulted in making camps either too big, and thus wasting labour and reducing the number of men per unit length of perimeter, or too small, causing overcrowding and confusion within the ramparts; a point made by Vegetius.<sup>34</sup>

**Camp Geometry**

If the camp surveyors laid out base rectangles of defined area and aspect ratio accurately, and not just approximately, they must have had a ready method of determining the lengths of the axes to be set out from the *groma*. It is certain that the method did not simply involve constructing empirical rectangles by adding together squares of one *a.q.*, like carpet tiles. This method could have produced only a limited number of base rectangles with areas and aspect ratios. Those within the required range of 12 to 285 *a.q.*, are listed in Table 3. Indeed, it is clear from tables 1 and 3 that most of the camps in this series, especially those of 4:5 ratio, *could not possibly* have been laid out by the “carpet tile” method.

TABLE 3

Areas of Rectangles Built up by the Carpet Tile Method:  
C and D comprise only whole numbers in *actus quadrati*D = *decumanus* C = *Cardo* A = Area in *actus quadrati*

Ratio 1:1		2:3			4:5			5:6		
C	A	C	D	A	C	D	A	C	D	A
4	16	2	3	6	4	5	20	5	6	30
5	25	4	6	24	8	10	80	10	12	120
6	36	6	9	54	12	15	180	15	18	270
7	49	8	12	96	16	20	320	20	24	480
8	64	10	15	150						
9	81	12	18	216						
10	100	14	21	294						
11	121									
12	144									
13	169									
14	196									
15	225									
16	256									
17	289									

It is almost certain, therefore, that a simple formula was used; one which took account of the variables of both area and aspect ratio. This was important because the same force might be required to make camps of different shape as the need arose; for example, on flat ground one night, on a small plateau the next, and on a hill top on the third. With square camps, the axis, or side, is simply the sq. root of the area but with oblongs another method is needed to find the axes. Vegetius<sup>35</sup> wrote, “. . . the surveyors should calculate the square footage of the site-plan so that the area enclosed corresponds to the size of the army”. The calculation requires a method, or formula, but Vegetius does not give it. Polybius says “. . . one simple formula for a camp is employed, which is adopted at all times and in all places”.<sup>36</sup> This paper suggests a formula which permits the calculation of the axes of the infinitely variable rectangles found in the bases of Roman camps and gives a unified theory to explain all the apparently whimsical rampart dimensions encountered, for example, the 2,017 feet of the camp of Polybius. It may be the formula to which Polybius refers.

A rectangle of aspect ratio 2:3 can be regarded as comprising of 6 equal subsquares laid together 2 by 3. Likewise one of 4:5 comprises 20 equal squares laid 4 by 5, see Fig 2. Thus the number of subsquares forming any rectangle is found by simply multiplying the numbers of the aspect ratio. Now, the length of the side of any subsquare is the sq. root of its area. For example, if its area is 16 sq. units, the side is 4 units long, since  $4 \times 4 = 16$ . It follows that the long side of the rectangle is a multiple of this sq. root value, 3 times in the case of a 2:3 rectangle and 5 times with a 4:5 one. We can say that the long side, or axis, of a 2:3 rectangle is 3 x the sq. root of one sixth of its area and the short axis twice that sq. root. Therefore the general rule to find the length of an axis in *actus* is:

Divide the area, expressed in *actus quadrati*, by the product of the aspect ratio numbers and obtain the sq. root of the answer. Then multiply this figure by the appropriate value of the aspect ratio.

Provided a table of square root values is available, the steps are quite simple. Suppose we need a 40 *a.q.* rectangle with a 2:3 aspect ratio, as at St Harmon.

$$\text{Then } 2 \times 3 = 6$$

$$\text{and } 40 \text{ divided by } 6 = 6.66$$

$$\text{sq. root of } 6.66 = 2.58$$

$$\text{The short axis is } 2 \times 2.58 = 5.16, \text{ or } 5 \text{ } \textit{actus} \text{ and } 19 \text{ ft. (619 ft.)}$$

$$\text{The long axis is } 3 \times 2.58 = 7.74, \text{ or } 7 \text{ } \textit{actus} \text{ and } 89 \text{ ft. (929 ft.)}$$

Another example is given by a 4:5 camp of 220 *a.q.*, Brampton Bryan.  $4 \times 5 = 20$ , and  $220 \text{ divided by } 20 = 11$ . The sq. root of 11 = 3.31, so the *cardo* is  $3.31 \times 4 = 13.26$  or 13 *actus* and 32 ft. (1,592 ft.), and the *decumanus* is  $3.31 \times 5 = 16.55$  or 16 *actus* and 66 ft. (1,986 ft.).

Returning to the case of the force needing to stop in three separate locations with camps of different aspect ratio because of topography, let us suppose it needs 120 *a.q.* On the first night, on flat open ground, it may dispose itself in the preferred 2:3 camp (Hyginus, Vegetius). The *cardo* is therefore  $120 / (2 \times 3) = 20$ . Sq. root  $20 = 4.47$ , and  $4.47 \times 2 = 8.94$ , or 8 *actus* and 113 ft. The *decumanus* is  $4.47 \times 3 = 13.41$ , or 13 *actus* and 49 ft. On the next night it occupies a position where a 2:3 ratio would straggle a steep slope and must therefore adopt a more square outline; let us

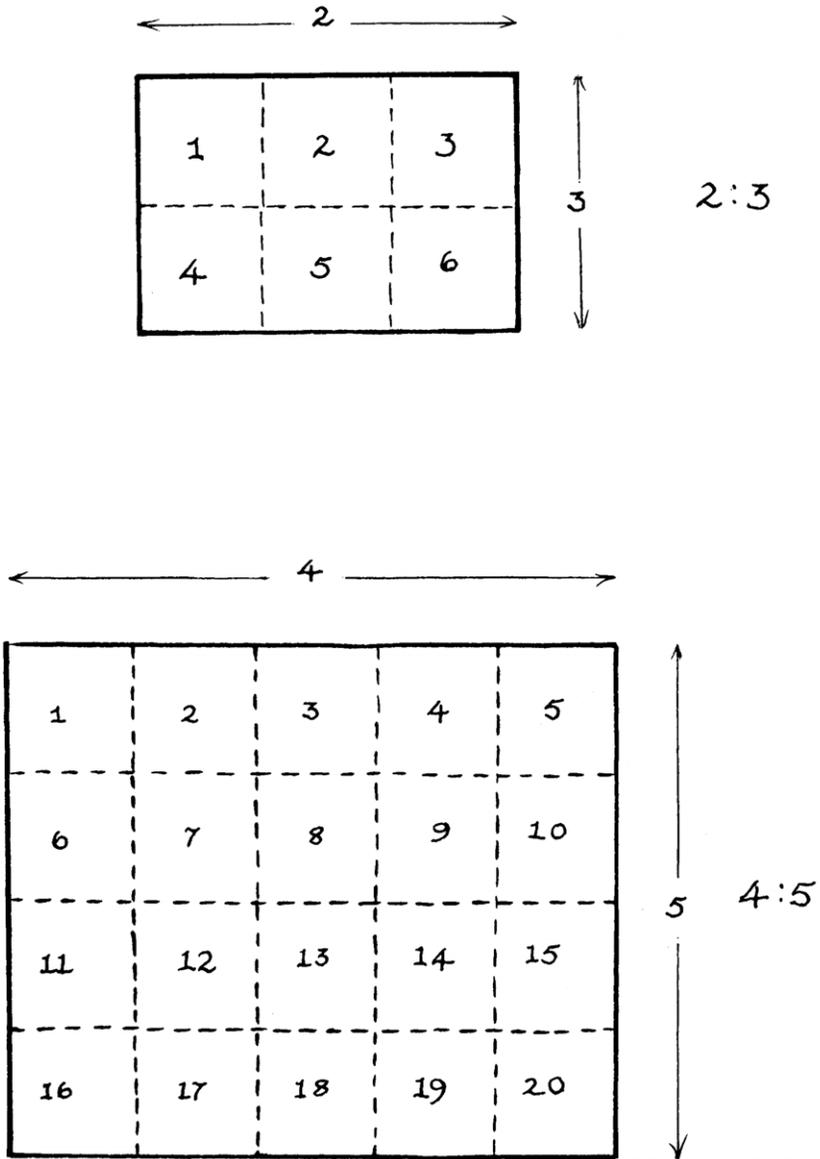


FIG. 2. Rectangles with subsquares.

say a 4:5 ratio will suffice. The *cardo* is 4 x sq. root (120/20) = 9.8, or 9 *actus* and 91 ft, and the *decumanus* is 5 x sq. root (120/20) = 12.24, or 12 *actus* and 24 ft. On the third night, the hill top demands a square configuration; so sq. root 120 = 10.95, or 10 *actus* and 102 ft.

The rule can be expressed algebraically. Where D = *decumanus* and C = *cardo*, in *actus*, A = area in *actus quadrati* and c:d is the aspect ratio, then,

$$D = d \sqrt{A/cd} \quad C = c \sqrt{A/cd}$$

The method, or something very like it, is *the only way to change the aspect ratio of a camp* of any given area, for example, the square 280 *a.q.* camp of Polybius to the similar sized 2:3 camp seen at Ardoch 6. It is an open question as to whether this is the precise formula to which Polybius refers.

According to Dilke, the "later" Greeks laid the foundations of algebra,<sup>37</sup> but prior to this, the Romans used somewhat laborious arithmetic to achieve similar results.<sup>38</sup> Clearly the sq. root values employed are critical, but there is no doubt accurate values were available. For example, Columella, who served in the army, had the sq. root of 3 accurate to 2 decimal places (1.73) when calculating the area of an odd-shaped piece of land.<sup>39</sup>

### Some Roman Camps in Cumbria

Table 4 lists details of certain symmetrical camps in Cumbria, described in the RCHM monograph<sup>40</sup> from whose plans the data were derived by simple measurement. It is noteworthy that the dimension of 1.97 *actus* occurs as the *cardo* in three small camps of different area and aspect ratio, Broomby Lane-2 (1:2), Barrockside (2:3) and Golden Fleece (4:5) and in the incomplete rectangle at Brougham. The dimension of 4.65 *actus* is seen as the *decumanus* at Broomby Lane-1 and in the incomplete rectangle at Beaumont.

TABLE 4

Dimensions of Some Roman Camps in Cumbria:  
(Lengths in *actus* and areas in *actus quadrati*)

Camp	<i>Cardo</i>	<i>Decumanus</i>	Area	Probable Area
1:2				
Broomby Lane-2	1.97	3.94	7.76	8
2:3				
Moss Side-1	2.53	3.38	8.55	8
Barrockside	1.97	2.96	5.83	6
Kirkby Thore-2	3.09	4.57	14.14	14
4:5				
Knowe Farm	3.17	3.94	12.48	12
Golden Fleece	1.97	2.53	4.99	5
5:6				
Kirkby Thore-1	5.53	6.76	38.07	38
Broomby Lane-1	3.94	4.65	18.34	18
9:10				
Moss Side-2	5.28	5.98	31.57	31
Almost Square				
Langwathby Moor	2.32	2.67	6.19	6

The camps at Rey Cross, Crackenthorpe and Plumpton Head, thought to be the work of the same force on its advance into Cumbria from Yorkshire, do not have any parallel ramparts. Their base rectangles were derived by averaging the lengths of opposite sides, see Table 5. These data are remarkable since the overall average rampart length is 8.58 *actus* or 1,030 Roman feet, giving a base square of 73.96 (say 74) *actus quadrati*, sufficient for a legion without auxiliaries (see below).

TABLE 5

Dimensions in *actus* of Camps at Rey Cross, Crackenthorpe and Plumpton Head:

Sides	1	2	Mean	3	4	Mean	Overall Mean
Rey Cross	8.67	6.88	7.75	8.66	8.27	8.46	8.12
Crackenthorpe	8.85	8.03	8.44	7.84	9.58	8.71	8.71
Plumpton Head	10.77	4.93	7.85	10.98	9.07	10.02	8.93
							8.58

## Discussion

To those with a maths. probia, these calculations might look forbidding but they are simple and could have been done on the spot by an averagely numerate officer with a short list of sq. root values. Squares roots were well known in antiquity, the Babylonians having calculated certain values,<sup>41</sup> and the Greeks likewise.<sup>42</sup> Despite Columella's value for sq. root 3, some other ancient values may not have been so accurate and may have produced some of the error observed in the camps listed in Table 1. But by simply transforming lengths in feet into *actus*, and area measures into *actus quadrati*, large area measures were handled simply. It is noteworthy that the fractions of *actus* found in this study are more-or-less simple fractions. According to Dilke,<sup>43</sup> by the late Republic, the Romans were using a special shorthand with a duodecimal system of fractions, i.e. based on twelfths. Thus a quarter was written as 3/12, a half 6/12 and so on. Now the ten-foot measuring rod, the *decempeda*, is 1/12 of an *actus*, so this arithmetic nomenclature was ideal for the setting out of lengths given in *actus* and 1/12 fractions thereof. It follows that two numbers describe the line; the first giving the number of *actus*, the second the twelfths. The base rectangle of any given area and aspect ratio is therefore described by just four numbers, two for each axis. The *actus* were probably measured with standard chains and the twelfths with *decempedae*. A 4:5 rectangle of 28 *actus quadrati*, as seen at Troutbeck, is given as D = 5-11, C = 4-9.

Given the relatively few possibilities required, it is more than likely that these dimensions were calculated by headquarters staff and set out in tables or learned, parrot fashion, by military surveyors. This was the preference of the late Prof. Eric Birley, when he commented on the author's drafts, and who was, as he put it, "familiar with the military mind". Indeed, with time, the dimensions needed for each type of camp would have become part of military folklore, to be quoted empirically by men who did not know what a sq. root was. Vegetius tells us that by his day, the late 4th century, the whole art and science of camp construction was lost.<sup>44</sup>

Future studies of Roman camps should consider the area of the base rectangle as well as the area within the ramparts, since this should give a closer correlation with the size of

the occupying force than camp area expressed in acres. Indeed, future studies might better illuminate this relationship. Dimensions should also be given in Roman units. The data given by Polybius suggest that a legion and its full complement of auxiliaries and cavalry required 140 *a.q.*, and operating alone about half of that, although these rules may have changed with time and circumstance. The base rectangles of 74 *a.q.* at Rey Cross, Crackenthorpe and Plumpton Head are consistent with a legion operating alone, or half a legion plus auxiliaries.

It seems the Romans adapted the surveying methods of agriculture to military needs and their fortified camp model met superbly the imperatives of action. It was sophisticated in concept yet was reduced to simple operating rules. The square camp was probably the prototype and in the Punic Wars was, perhaps, the only model. It is possible to imagine the evolutionary steps which took the square to the oblong, while maintaining the same area and, most importantly, internal organisation. If a flat, square area were not available, but an elongated one was, it would be logical to make 2 squares of half the area and lay them together. This, in effect, is a 1:2 rectangle comprised of two subsquares. The other rectangles are a logical extension of this model and the parallelograms seen near Numantia show that it was in use by the 140s BC.

The mathematical method demonstrated above gives the axes in *actus* and feet, or, more likely, tens of feet, and the order to the men would be very clear, “*x actus, y decempedae*”: once the ground was chosen, the task was straightforward. Knowing the size of the force and its required area, and having decided the aspect ratio, the surveying officer, *agrimensore*<sup>45</sup> could consult his pocket tables and set out the axes, perimeter and internal grid with speed and confidence. A civil engineer, experienced in setting out large areas, has informed the author that it would have been important to check that the corners were square, because an error of half a degree in the *groma* would give an error of 4.36 metres at 500 metres. Ideally, one would start at one side, but a smaller, exact rectangle of the desired aspect ratio could have been first set out in the middle and then its long axis extended to the required length of the camp *decumanus*. The corners of the larger rectangle could then be fixed by sightings on the corners of the smaller rectangle and the ends of the *decumanus*. The theorem of similar triangles, well known in antiquity, ensures that the aspect ratio of the large rectangle is the same as the smaller. This method would require only the *decumanus* to be calculated, or memorised.

One can well imagine the *mensores* jumping-to at the bark, “*Decumanus 10, 3!* – at the double!” Within a short time, perhaps half an hour, a 2:3 rectangle of 70 *a.q.* for a legion is pegged out, and the arriving troops, knowing precisely their place within the *strigae*, fall to their assigned duties almost without orders. The whole ritual of construction was doubtless a psychologically bonding and morale lifting exercise. It could also be a terribly demoralising spectacle for the enemy, especially uncivilised barbarians. Imagine, on the eve of battle, with what emotion they would witness this awesome machine create, in the hour before sunset, a veritable city from the wilderness.

### Acknowledgements

I wish to acknowledge the help of Dr B. Dobson who kindly supplied the translation of Hyginus, and to the late Prof. O. A. W. Dilke and the late Prof. E. Birley for constructive criticism. Mr T. W. Richardson commented on setting out a rectangle.

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- <sup>38</sup> Dilke, O. A. W., *The Roman Land Surveyors* (Newton Abbot, 1971) 54-57.
- <sup>39</sup> *Ibid.*
- <sup>40</sup> Welfare, H. and Swan, V., *op. cit.*, 30-52.
- <sup>41</sup> Dilke, O. A. W., *Reading the Past*, 11, 20.
- <sup>42</sup> Dilke, O. A. W. personal communication.
- <sup>43</sup> Dilke, O. A. W., *Reading the Past*.
- <sup>44</sup> *Vegetius, op. cit.*, 23.
- <sup>45</sup> *Ibid.*, 80.

