ON THE PROGRESS AND PROSPECTS OF SCIENCE IN SCOTLAND AT THE CLOSE OF THE SIXTEENTH AND COMMENCEMENT OF THE SEVENTEENTH CENTURIES, AS COMPARED WITH THE SAME AT CAMBRIDGE A CENTURY LATER:

WITH ILLUSTRATIONS OF SEVERAL REMARKABLE COINCIDENCES BETWEEN THE GENIUS, THE STUDIES, AND THE DISCOVERIES OF NAPIER OF MERCHISTON, AND SIR ISAAC NEWTON.[1]

BY MARK NAPIER, ESQ.

I HAVE been honoured with a request to contribute a memoir touching the antiquities of science in Scotland as compared with its condition at Cambridge of a much later period. Not that I have the slightest pretensions to be considered scientific, but the command of some original documents among the family archives of Napier of Merchiston, the inventor of logarithms (the only philosopher who illustrates Scotland in the great era of Tycho, Kepler, and Galileo), and a closer attention bestowed by myself than by any one else upon the habits and history of this great Scottish worthy, may perhaps enable me at least to amuse, if I do not instruct, the learned whom I have the honour to address.

If Mr. Macaulay be right in his estimate of Scotland, even at so late a period as the commencement of the XVIIth century, when our Sixth James migrated, nothing loth, to more abounding England, the less we look into our social antiquities, and the more we dwell upon our scientific, the less cause shall we find to blush for our ancestors. That gifted historian, whose mode of announcing new facts is as fearless as it is brilliant, speaking of the comparatively modern era when the union of the crowns had placed the resources of three kingdoms at the command of one monarch,

[1] Communicated to the Historical Section, at the Annual Meeting of the Archæological Institute in Edinburgh, July 25, 1856.

contrasts the condition, intellectual and social, of Scotland with that of Ireland at the same period. Taking his readers by storm with one of his rapid and dazzling generalisations, he thus issues his fiat as to the leading characteristics of Scotland at the commencement of the XVIIth century, the grand era of science :—

> "In mental cultivation (he says) Scotland had an indisputable superiority. Though that kingdom was then the poorest in Christendom, it already vied in every branch of learning with the most favoured countries. Scotsmen, whose *dwellings* and whose *food* were as wretched as those of the Icelanders of our times, wrote Latin verses with more than the delicacy of Vida, and made discoveries in science which would have added to the renown of Galileo. Ireland could boast of no Buchanan or Napier."

We must be allowed to doubt the historical accuracy of this elegant and laboured antithesis. We are not aware that any such extreme discrepancy between social resources and intellectual powers ever existed in any age or country. We cannot believe that it was the case in Scotland at the commencement of the XVIIth century. We will not accept the compliment, even from Mr. Macaulay, at the expense of his banter. An archæological excursion through Edinburgh, indeed through Scotland, under such accomplished guides as a Daniel Wilson or a Robert Chambers, would have been no less instructive to our prime historian than would have been a lecture on the Roman remains, bestowed upon our Prime Minister before the Crimean campaign, according to the intelligent suggestion of Dr. Bruce. But the dramatic historian of England, ever fond of pointing his moral and adorning his tale with an illustrious name, has not failed to peril his proposition upon individual instances. We accept his challenge, then, under the special examples offered. There is no reason to believe that Master George Buchanan, who certainly wrote Latin verses with more than the delicacy of Vida, was ever at a loss for a comfortable lodging and a good dinner. Indeed, he dwelt very much in a palace ; and many must have been the regal tit-bits, the savoury crumbs of pasties and preserves, the savoy-amber, the pistache amber, and the fennel, that adhered to the liquorish moustache of the royal dominie.

The instance is no less unfortunate as regards the wealthy laird of the Logarithms. He possessed various dwellings all over Scotland, from besouth the Forth to benorth the Tay; and every one of them most substantial. Many and changeful were the characteristics and conditions at different times of the old Castle of Merchiston; but, assuredly, Icelandish it never was. The Scottish worthy, whose scientific genius Mr. Macaulay so fully appreciates, was, to say the least, as commodiously housed, and doubtless was a more regular and comfortable diner, than Sir Isaac Newton at Cambridge a century later. Napier was a great store farmer. He was careful of stock, and curious in cultivation. This Icelandish Scotsman's time and great genius were about equally bestowed upon the mysteries of Agriculture, of Algebra, and of the Apocalypse; and we doubt not he would have grimly chuckled over so figurative a description of his "dwellings" and his "food" as that with which we are favoured, *currente calamo*, by the most popular writer of the XIXth century. "Merchiston's new order of tillage and pasturage," and especially his instructions for the management of the milk-cows on the home-farm, so that they might give double the quantity of rich milk—a system of home-farming set down by himself so early as 1598—suggests no idea of Iceland, as we peruse the placid and pastoral record. Cuyp might have painted from it; and the quaint beards that for generations wagged merrily in those old halls, had grown out of the best of beef and Easter ale, besides "wild meat," as game was then designed, comfits, "fine hetted kit," and "chopins of claret wine," long before the time when, says Mr. Macaulay, the intellectual immortality of Scotland dwelt wretchedly in Icelandish huts, and fed on garbage!

But I must not allow this tempting text, although really susceptible of some very curious illustrations to its complete discomfiture, to allure me from the particular subject of the present paper, which belongs to the archæology of science. I propose to look back upon those picturesque times, when the chrysalis of the adept was still hanging upon the brilliant wings of science—when astronomy had not yet escaped from judicial astrology, nor mathematics from magical squares and the mysterious powers of the numbers five and seven, nor chemistry from the alluring promises of faithless Hermes. My purpose is, so far as time will permit, to compare Scotland of that period with Trinity College, Cambridge, a century later. What was doing anent science and philosophical matters in Icelandish Scotland during the XVIth and XVIIth centuries, at a time when Newton was uncreated dust?

From private papers, as well as from the published records of science, it can be shown that the advent of Newton was being there typified; the way was being there made straight for him, even in what may be called the wilderness of science, a century before he came. The remarkable coincidences between the studies, the discoveries, and the genius of Napier and of Newton, have not attracted even in Scotland that attention which a fact so interesting to the intellectual fame of our country deserves. It can be shown that Napier had surveyed the whole field of Newton's triumphs with a curious anticipation, indicating a bent of genius singularly coincident with his in all its phases; that he had actually bequeathed both the principle and the nomenclature of *Fluxions;* that as regards alchemy, the searching for the hidden treasures of the earth, and the practical details of the royal mint; arithmetic and algebra; mechanics and catoptrics; the curiosities and refinements of domestic agriculture; and the sacred mysteries of the Prophet Daniel and the Book of Revelations,—Napier trod in the very paths, and with no tottering steps, where the march of Newton so majestically followed a century after. These coincidences, indeed, are so striking as to justify the figure, that the antique mirror of the King of Numbers reflected the coming form of the Prince of Mathematicians.

I. Coincidence in their pursuit of Alchemy.

I commence the comparison with the state of Alchemy in Scotland during the XVIth and XVIIth centuries, as compared with the same at Cambridge, in the hands of Newton, a century later. Even that subject cannot be fully discussed upon this occasion; and I must limit myself, as regards Scotland, to a few illustrations derived from contemporary manuscripts, which have never yet appeared in print.

The first of Napier's manuscripts to be submitted to this assembly, discloses, in graphic terms, a very curious scene, occurring in Edinburgh precisely two hundred and forty-eight years ago. As the context proves, it was carefully recorded at the time by Napier himself; but the manuscript was lost sight of, and has been too recently recovered to have been entered in his biography. It need only be further premised that the philosopher, having been installed in the fee of the barony of Merchiston before his father's death, invariably subscribed himself, while his father lived, "John Napier, fear of Merchiston:"—

"Upon Saturday, the 7th of November, 1607, I, John Napier, fear of Merchiston, came to confer with Mr. Daniel Muller, Doctor of Medicine, and student in alchemy, anent our philosophical matters. Not knowing that he was sick, and finding that he was diseased of the gout, his ordinary disease, I thought not to have troubled him with much conference, and meant to have left him for that time; but he, craving conference of me, showed me that he was to have sent for me if I had not of accident come, and that he had a matter to communicate with me, if I might then remain, or shortly return. So I removed my company, and sat down before his bed-side. Then he burst forth in these words:—

"'Sir, you are occupied in alchemy: I have been, these many years, a very earnest student thereinto, and have attained to the knowledge thereof. I have pressed to have diverted you from your wrong opinion, so far as I durst be plain. But now, sir, I will be plain, knowing that you are a man who fears God, and will be secret; and that you will be good to my wife and bairns, in case these diseases shall take me away.

"'Sir, I sent a credible friend to Histria (a Venetian province at the top of the Adriatic), to bring me hither of crude mercury out of those mines a long time since, and as yet I have heard no word from him; I think he be dead. I once received a little piece of the earth of those mines, about the quantity of an hazel nut, which, as I brake, there appeared scales of quicksilver within the same, and the crude mercury flowed forth without the fire. With this I perfected the philosophical work, as you may do with the like: for this mercury, being taken with fine silver which never did find fire, and inclosed in a matrix, will become black within the space of forty days, and thereafter will become white, and then is the point and term to loose it, if you do not join it with fine gold that never did find the fire, when instantly that which was taken of mercury and *luna*, or silver, will devour up the gold; and at this conjunction or fermentation endeth the first work, called *opus lunæ* (the silver operation), and beginneth immediately the second, called *opus solis* (the golden operation).

"'In this *opere solis* your work becomes blacker than *in opere lunæ*, and then white, and at last, red.

"'Both these works are performed in a year—to wit, two months and a-half *in opere lunæ*; and nine months and a-half *in opere solis*.

"'And for *pondera* I take nine of crude mercury to one of crude *luna* (or silver), *in primo opere*; and this I conjoin with one of *sol* (or gold) *in secundo opere*.

"'So *luna* is the *medium conjungendi*; and hereof cometh three mercuries—to wit, the first, which is *mercurius crudus*, and is called *mercurius frigidus, acetum, mercurius mineralis*; the second, which is *luna* dissolved in crude mercury to the point of whiteness, is called *mercurius tepidus, acetum acerrimum, mercurius vegetabilis, quia luna est planta* (because silver is the root) ; the third, which is *sol* dissolved by the second, is called *mercurius calidus, mercurius animalis.*

" ' Further,' said he (Dr. Muller), ' the little cipher table, entitled " Medulla Philosophiæ Hermetriæ," it is mine, for I made it.'

" Also, he added many discourses, citing texts out of Clangor Buccinæ, Marsilius, Ripleus, and Arnoldus, to prove the premises, and especially ' De Terra Nigra Occulosa, Terra Hispanica,' &c.

" Further, he said that the various hued glass which I did see was in that manner, throughout all its texture, coloured with the stuff which he made in that same glass.

" Further, he spake to the *triplici usu lapidis,* after Paracelsus—first, in transmutations of metals ; secondly, in curing diseases ; and thirdly, it is *lapis divinus,* for magical uses.

" Now, when I heard these things, and had said unto him, ' My lord, that matter is marvellous, if you be sure of the truth thereof by practice,' he answered with earnestness, ' In truth I have practised it to the end, and made projection and found it true.'

" Again, when I demanded of him, how it fortuned that he did not multiply his stuff, and keep the same, he answered, ' I lacked crude mercury, without which it cannot be multiplied again.'

" Upon the 9th of November, I conferred with him again anent some doubts, *quod fons trahit regem, et non rex fontem,* and so doth *aqua-regis ;* but vulgar mercury, on the contrary, *non trahit solem, sed sol eum ?* He answered, that whatever vulgar mercury or crude mercury do, yet this mercury philosophical, of crude mercury and silver, will instantly drink up gold, and draw it in, *initio secundi operis.* Then I demanded, when should the second work begin, and what was the sign before the point of danger to the work ? He answered, that after perfect whiteness *in opere primo,* there would appear, in an instant, a small hair-like circle surrounding the matter, and attached to the sides of the vessel ; then instantly ferment with gold, and it will presently eat up all the gold, and that circle will vanish ; but, if you stay longer in fermenting, the work will become all *citrine,* and more dry than that it can dissolve the gold ; for the gold must be sown *in terram albam foliatam.*

" Then I demanded what *terra alba foliata* was ? He answered, that at the point of whiteness, in the first operation, the matter of mercury and *luna* became like the small scales of a fish. Then I remembered that my father showed me that he made a work which became *terra alba foliata,* most like the leaves of a book set on edge, of *sol luna, aqua-regis,* and *aqua-fortis.*

" Upon the 13th day of November, he, being convalesced, showed me that he had feared himself (thought he was dying), and out of affection had revealed these things to me, which, upon his salvation, he affirmed to be true, and desired me to confer the sentences of the philosophers together, and I should find them all agree with these premises ; which I find apparently very true in their theoretical sentences ; but, on the contrary, in their practical precepts, they induce many things repugnant to themselves, to illude the vulgar and profane people, and to divert them from the truth of their former sentences.

" Thereafter, about the 15th day of March, 1608, the Doctor showed me that he had received glad tidings of the safe return of Lionel Struthers, his said friend, from Histria to England ; and he showed me a certain antique figure, with certain verses of congratulation which he had made, and was sending to him in joy of his safe return.

"So, within ten days, he came to Edinburgh to the Doctor, and brought with him great store of mineral mercury, which never had felt fire, and some unfined, easy to be wrung out from his ore. The Doctor gave me, secretly, a small portion both of the one and of the other; as, also, a very small part of *luna* mineral unfined; but I purchased more, both of Scots and German *luna*. As for *sol* (gold) mineral, we have enough in Scotland, rests time and opportunity to enterprise the work, with the blessing of God to perform the same, to his glory and comfort of his servants, which the Almighty grant to us, whose holy name be praised and magnified, for ever and ever. Amen.

"Mr. Struthers says that the Spaniards take all the said crude mercury, for it gathers most of mine gold."

This curious document enables us to institute a comparison in the matter of alchemy, between the author of the Logarithmic Fluxions, and his great antitype, the author of the "Fluxionary Calculus." From it we may gather that Napier, even in his remoter age and ruder country, was, to say the least, as cautious and sceptical in his reliance on the adept, as was Newton in his riper epoch, at Cambridge. Let us then take a walk, a century later, in Trinity College, Cambridge, that we may not too hastily condemn or deride such investigations as "follies of the wise."

It has rather taken the world by surprise to learn, of late, that Sir Isaac Newton was an Alchemist. The fact may tend to elevate our notions of that exploded and explosive study, and of the minds and motives of those men of genius, who wasted the midnight oil, and their daily bread, in endless efforts to present us with a stone. But it never can reduce our estimate of Newton.

In the first edition of the best biography of him, Newton's devotion to alchemy was not sufficiently known, and therefore not conceded. "There is no reason to suppose," said his gifted expounder and eulogist, "that Sir Isaac Newton was a believer in the doctrines of alchemy."

The recent greatly expanded edition of that valuable biography has shed a broader and less dubious light upon a curious and hitherto unobserved phase of England's greatest mathematical mind. Original letters, contained in various publications, have added their stores to the previous researches, and our own revered prophet of light has been constrained to submit to the perhaps unpalatable duty of disclosing his illustrious subject, Sir Isaac Newton, with his conjuring cap on.[2]

Accordingly, we are now told—"Newton, at one period of his life, was a believer in alchemy, and even devoted much time to the study and practice of its processes."[3] But the period of his life, when he was thus too much engrossed by labours comparatively, though not entirely fruitless, comprehends, we find, no less than about thirty years of the best period of his mental and bodily vigour. And, however his faith may have become latterly somewhat shaken in the omnipotent capabilities of the crucible, no evidence appears that he ever absolutely renounced his long allegiance to Hermes Trismegistus, King of Thebes, and great-grandson to Noah.

In the year 1669, writing to a young friend, Mr. Francis Aston, on the eve of his travels, among various instructions how to improve the occasion, the most earnest seems to be the following:—"Observe the products of nature in several places, especially in mines; with the circumstances of mining, and of extracting metals or minerals out of their ore, and of refining them; and, if you meet with any transmutations out of their own species into another, above all those will be worth your noting, being the most luciferous, and many times lucriferous experiments in philosophy."[4]

This interesting letter contains many other instructions relative to observing all the processes of angling for gold with mercury, throughout the mountains and streams of Hungary, Sclavonia, and Bohemia; and there is even an anxious injunction, imparting somewhat of an Arabic air to this instructive missive, that his Telemachus should be on the look out for a certain individual in Holland. "I think," writes Sir Isaac, "he usually goes clothed in green, and was imprisoned by the Pope, to have extorted from him secrets of great worth, both as to medicine and profit, but escaped into Holland, where they have granted him a guard."

There is no mistaking this language. It obviously emanates from a mind teeming with hermetic aspirations, and from one whose very soul was saturated with *mercurius crudus*, *sol*, and *luna*.

[2] Sir Isaac Newton: Memoirs of his Life, Writings, and Discoveries. By Sir David Brewster, K.H. Two vols. 8vo. Edinburgh, 1854.

[3] *Ibid.*, vol. ii., p. 371.

[4] *Ibid.*, vol. i., p. 388.

True, Sir Isaac at this time was only in the twenty-seventh year of his age. But for eight of those years he had been a distinguished student at Trinity College, Cambridge; was already deep in Descartes, and, indeed, had passed the period of his first conception of the fluxionary calculus. Nay, the letter from which we have quoted is dated three years subsequent to that pregnant occasion when he noted the fall of the famously suggestive fruit, which thus became the second memorable apple in the history of mankind.

About sixteen years after this advice to the young traveller, we discover the coming glory of England, instead of being reclaimed from these "follies of the wise," occupying, like another Sidrophel, the centre of his magic circle. Between the years 1683 and 1689, he is graphically presented to us, by his assistant, Dr. Humphrey Newton, as for ever flitting round a furnace in his laboratory—" the fire," says the Doctor, " scarcely going out either night or day; he sitting up one night and I another, till he had finished his chemical experiments, in the performance of which he was most accurate, strict, exact. What his aims might be, I was not able to penetrate into; but his pains, his diligence, at these set times, made me think he aimed at something beyond the reach of human art and industry." [5]

In another letter, Dr. Newton becomes a little more explicit. "About six weeks at spring," he tells us, " and six at the fall, the fire in his laboratory scarcely went out; which was well furnished with chemical materials, as bodies, receivers, heads, crucibles, &c., which were made very little use of, the crucibles excepted, in which he fused his metals. He would look sometimes, though very seldom, into an old mouldy book which lay in his laboratory; I think it was titled 'Agricola de Metallis;' the transmuting of metals being his chief design."

But Sir Isaac did more than dip into that one old volume; he absolutely pastured upon the voluminous records and rankest grass of the kingdom of Trismegistus. The jargon of that mysterious potentate's disciples could never have been out of Newton's head; and their hieroglyphic signs must have been for ever dancing before his prismatic eyes, like motes in the beams of the sun, or spots upon his

[5] Sir Isaac Newton: Memoirs of his Life, Writings, and Discoveries. By Sir David Brewster, K.H. Vol. ii., p. 93.

disc. "There exist," his modern biographer tells us, "many sheets in Sir Isaac's own writing of Flamel's 'Explication of Hieroglyphic Figures,' and large extracts out of Jacob Behmen's works." "We have seen," he adds, "in Sir Isaac's handwriting, 'The Metamorphoses of the Planets,' by John de Monte Snyders, in sixty-two pages quarto, and a key to the same work; and numerous pages of alchemist poetry from Norton's 'Ordinal,' and Basil Valentine's 'Mystery of the Microcosm.' There is also a copy of 'Secrets Revealed; or, An Open Entrance to the Shut Palace of the King,' which is covered with notes in Sir Isaac's hand, in which great changes are made upon the language and meaning of the thirty-five chapters of which it consists." "I have also found," continues his biographer, "among Sir Isaac's papers, a beautifully-written but incomplete copy of William Yworth's 'Processus Mysterii Magni Philosophicus;' and also a small manuscript in his hand-writing, entitled, 'Thesaurus Thesaurorum Medecina Aurea.' In addition to these works, Sir Isaac has left behind him, in his note-books and separate manuscripts, copious extracts from the writings of the alchemists of all ages."

From another original and unprinted manuscript, yet preserved in the Napier charter-chest, written subsequently to the death of the inventor of Logarithms, by a younger son, but before the birth of Newton, some idea may be formed of Sir Isaac's purpose in submitting his great mind to the endless toil of extracting these barbaric authors, even including their most execrable poetry, for the prompting of which Apollo ought to have kicked Mercury round the circle of the heavens.

Faith in alchemy seems rather to have increased than diminished during the century that separates Napier from Newton. The son of the Scottish philosopher had toiled in the vineyard of Trismegistus far more devotedly than his somewhat sceptical father; and yet, even he would seem to have been idle as regards both the study and practice of alchemy, by comparison with the thirty years of labour, mental and manual, submitted to by England's greatest mathematical mind.

Robert Napier, however, to whose manuscript I am about to call attention, by extracting the marrow of all the hermetic philosophers and authors who preceded him, was thus enabled, as he imagined, to separate their truthful doctrines and precepts from their wilfully deceptive mystifications; and he actually bequeathed to his son that which he does not profess to have received from his own father, namely, the grand secret itself.

I hold the precious gift in my hand. But the paternal blessings with which the awful boon is announced are so blended with anathematizing, and the pure worship of God, which the preface inculcates, is so closely allied to the most exclusive worship of Mammon, that I have scarcely ventured, as yet, beyond the limits of its lunar preface, into the solar realms of the opened palace of the king, that lie beyond. Nor, I believe, has any one but myself ventured to master even the preface.

The first injunction is written in such English as was then commanded by its profound author, Master Robert Napier of Culcroich, Drumquhannie, and Bowhopple.

"This book is to remain in my charter-chest, and not to be made known to any, except to some near friend, being a scholar, studious of this science, who fears God, and is endowed with great secrecy, not to reveal or make common such mysteries as God has appointed to be kept secret among a few, in all ages, whose hearts are upright towards God, and not given to worldly ambition or covetousness, but secretly to do good, and help the poor and indigent in this world, as they would eschew the curse of God if they do otherwise.—R. NAPIER."

But the title, the caveat, the preface, and the treatise itself, are all in Latin, which I must take upon me to translate only to the very limited extent that cannot put any of my present hearers in possession of the secret which this *libellum* contains. He calls it—

"The revelation of the mystery of the Golden Fleece; or a philosophical analysis, whereby the marrow of the true hermetic intention is made manifest to such of my posterity as fear God.—ROBERT NAPIER, author."

Then comes this solemn caveat:—

"Beware that you do not make public this little book to the impious, the imprudent, or the garrulous. Beware!"

After which follows the preface:—

"My beloved son,—And be thou initiated as a son of this art and in the principles of this sacred science: above all things, seek God with your whole heart, and embrace him with a pure spirit; for without the guidance of God all is vanity, especially in this Divine science, which, even from the Deluge down to these times, the Almighty hath been pleased to reveal only to a very few, and these good men, and gifted with Pythagorean silence. God, the searcher of hearts, directs both the mind and the hand. He bestows this science when he wills, and upon whom he wills. And it is not his will that pearls should be cast to swine (*nec margaritas porcis projici voluit*). Whoever divulges these sacred mysteries, shall be held guilty of betraying his secret, and responsible for all the ills that may emanate therefrom. A madman must not be armed with a sword. Divulge this secret, and the hind would become greedy of gold to his own destruction. Iniquities would cover the earth ; agriculture and the other arts of civilization would no longer exist. Mighty in their gold, nations would rush to war for nothing. The worthless would wax proud, and scorn their rulers. The reins of civil power and legitimate government thus relaxed, a fearful earthquake would follow. Oh ! I say, reveal this secret to the vulgar, and the darkness of chaos will again brood upon the face of the waters.

"But that all knowledge of so great a gift of God might not perish, and that the wise and the good might, even in this mortal life, obtain a foretaste of the supreme goodness of God to his own glory, it has been ordained by Divine Providence that this science should be transmitted to us, from Hermes, its first inventor, down even to these times, a period of nearly 4000 years, through the hands of the learned—the majesty of the great mystery being protected in a cabalistic form. That such a science exists, has also been made known to us through books ; but these, for the most part are so full of enigmas, allegories, and figures of speech, nay, of falsities, mystifications, and contradictions, that they seem rather to have been written for misleading than for instructing. Long would be the time, and weary the wandering in error, ere this divine art could be acquired by any one from the books of the philosophers, without a faithful guide.

"But I, my son, moved by paternal care and affection for you, and towards all of my posterity who serve the living God, lest seduced into error by these books you waste the precious time in vain, and fruitlessly expend both money and labour in search of this divine art, for your sake have determined in my own mind to treat of the art truthfully, plainly, and systematically, by collecting together in this manuscript all the most trustworthy sentences of the philosophers which I find confusedly scattered throughout their many books, and to digest them in methodical order. And this I have undertaken that you, thus rendered competent and learned, both as regards the process and the material, and grateful for so great a gift of God, may direct it all to his glory by exercising beneficence to the poor, by relieving all their wants, and alleviating all their bodily sufferings.

"First, however, I adjure thee, and whomsoever of my posterity may happen to see and read this manuscript, by the most holy Trinity, and under the penalty of the Divine vengeance, that you publish it not, and make it known to no one, unless he be a son of this art, a God-fearing man, and one who will keep the secret of Hermes under the seal of the deepest silence.

"If you do otherwise, accursed shalt thou be ; and, guilty before God of having betrayed his secret to the wicked, most assuredly the Divine vengeance will light upon your head for all the evils that may thence arise.

"May my own soul be free from so deadly a sin. My constant prayer to him is, that this manuscript of mine may by no accident fall into impious hands. And I here call Him to witness, that it was collected and written by me solely for the sake of good men, who with sincere and pure hearts worship God, to whom be all honour, praise, and glory, for ever and ever."

We now know that this mouldy manuscript, likely in these days to be perused, or rather glanced at, with a smile not only of credulity but of compassion, would have found favour in the sight of Sir Isaac Newton. For, dipping cautiously into the revelation of the golden mystery itself, the very first philosopher whom we find quoted is, "Flamelli Hieroglyphica," or Flamel's explication of hieroglyphic figures, of which many sheets have been discovered among Sir Isaac's papers, in his own handwriting. We also find extracts from "Nortonus Anglius," and "Basilius Valentinus;" being the same authors, doubtless, as those mentioned by Newton's biographer, when he tells us, that he has "seen in Sir Isaac's handwriting numerous pages of alchemist poetry, from 'Norton's Ordinal,' and 'Basil Valentine's Mystery of the Mycrocosm.'"

Neither has our curiosity been disappointed, in searching through this manuscript for some notice of that alchemist friend of the inventor of logarithms whom we have already discovered labouring under the double agony of gout and gold. When, in Robert Napier's manuscript, we found a quotation from "D. D. Mollierus," we could not doubt that this means "Doctor Daniel Muller." It will be remembered that this worthy sent an antique figure, with some congratulatory verses, to hail the happy return from Istria of his friend "Lionel Struthers," who in these comparisons may be taken as the pendant to Newton's young travelling friend, Francis Aston. And we are happy to be able to present a specimen of that alchemist poetry which Sir Isaac delighted to transcribe. Robert Napier favours us both with the original in Latin, and with an English version by himself.

"D. D. MOLLIERUS.

Clavicula triplici proprio de stemmate facta,
Ingenue reseror: quarum jacet una sepulta
Monte sub Istriaco; Mariana Monte secunda;
Tertia soliferis Scotiæ reperitur in undis.
His tribus unitis cedo non viribus ullis;
Longævus, sanus, locuples, reserator abibis."

"ENGLISHED THUS:

A three-fold key soon opens me, made of my proper kind;
The first lies still in Istria Hill, there buried in that mine;

The next is wont in Marian Mount to lie among the mould ;
The third is found in Scottish ground, in waters breeding gold ;
This units three does open me, I fear none other force ;
Depart with wealth, long life and health, thou opener of my corse."

At the very time, however, when laboriously studying such poetry as this, the mighty mind of Newton was giving birth to the "Principia Mathematica!" And when, nearly an hundred years before, Napier was discoursing at the bedside of Dr. Daniel Muller, about Mercurius, Sol, and Luna, he had the Logarithms in his pocket, though not given to the world until six years thereafter! As for Newton, while thus painfully sacrificing at the altar of Hermes, forbearing to sleep, forgetting to eat, disdaining to sit, and all in search of the golden Fleece, his immortality had already responded to the call both of Napier and of Kepler. Wielding Napier's great discovery, namely, the logarithmic principle and power of progressions and relative proportions or ratios, deeply indebted, both arithmetically and algebraically, to the Naperian canon of the Logarithms, in which that teeming principle was for the first time developed, and completely armed for practice, Newton, with the advantage of a new and powerful algorithm, continued to expand and fructify this most suggestive Institute of Numbers, through the binomial theorem, into the boundless region of transcendental algebra. Seizing, with tenacious grasp, the great law of the heavenly bodies, which had been so opportunely promulgated by Kepler, he conceived and completed the demonstration of universal gravitation.

Newton has compared himself and his discoveries to a child picking up pebbles on the shore of a vast unexplored ocean. Yet he did not actually pick up the most precious of his pebbles. They were presented to him by the children of a previous century, who were far from unconscious of the value and latent virtues of their rugged gifts. But his was the destiny to cut and polish those precious pebbles, until the face of nature became reflected therein.

The coincidences, however, between the genius of Napier and of Newton, in the higher and more approved departments of science must now be illustrated.

II. Coincidence of their Commentaries on the Prophecies.

Sir Walter Scott, speaking from a very imperfect consideration of the circumstances, observes in his "Provincial Antiquities":—"The sublime genius which marked by the Logarithmic Canon the correspondence betwixt arithmetical and geometrical professions, had his weak point. Napier, like Newton, wasted time in endeavouring to discover the mysteries of the Apocalypse, and to ascertain prophecies, which, if intended for our instant comprehension (with deep respect we speak it), would have been expressed more clearly."

The degree of weakness, however, if weakness it be, in this matter, is very different as regards these great intellects. The rude and unenlightened condition of theology, and the unsettled and alarming prospects of Protestant Europe, and of the British Isles in particular, when Napier put forth in haste (expressly to meet a great crisis) his long-considered theological work in the year 1593, must redeem his undertakings from the imputation of mere weakness. His was the first great theological work of the kind; and neither from the hands of Sir Isaac Newton, nor of Dr. Newton, or Mede, or the whole host of Apocalyptic commentators down to the present day, has the world obtained a treatise more exhaustive of the hopeless subject, or one which for originality, ingenuity, and profound and varied erudition, can stand a comparison with Napier's "Plain Discovery of the whole Revelation of St. John." Why Sir Isaac Newton, in his own elaborate and earnest, but, by comparison, rambling disquisitions on the same subject, entirely ignores Napier (as he also does in science), is a question I cannot answer. But the "Plain Discovery," although a voluminous digest, had been translated before Sir Isaac's time, into Latin, French, Italian, and German; and would have been the very best institute on the subject to which he could have attached himself. Doubtless the sceptic Voltaire said it with an unseemly sneer, and careless of ingenious and fruitless varieties, but he was substantially well-founded in the remark, that the prophet of universal gravitation added nothing whatever to human triumphs in this unconquered field, but "explained the Revelations in a manner very similar to all the commentators who had preceded him."

Napier's theological studies arrived at their culminating point under very peculiar circumstances. We have evidence from himself of the intense working of his speculative mind at the early age of not more than fourteen years. The greatest *alumnus* ever reared by the *Alma Mater* of Scotland, he matriculated as a student of St. Salvator's College of St. Andrews in 1563, being then not fourteen years old. This was only three years after the Parliamentary establishment of the reformed doctrines, and St. Salvator's was still remarkable for the divided state of religious opinion. In his address "to the godly and christian reader," prefixed to the "Plain Discovery," he himself affords this graphic account of the earliest energies of a mind destined to create a great revolution, not in religion, but in science.

"In my tender years and bairn-age in St. Andrews at the schools, having on the one part contracted a loving familiarity with a certain gentleman, *a papist*; and, on the other hand, being attentive to the sermons of that worthy man of God, Master Christopher Goodman, teaching upon the Apocalypse, I was so moved in admiration against the blindness of papists that could not most evidently see their seven-hilled city, Rome, painted out there so lively by Saint John, as the mother of all spiritual whoredom, that not only burst I out in *continual reasoning* against my said familiar, but also, from thenceforth I determined with myself, by the assistance of God's spirit, to employ my study and diligence to search out the remanent mysteries of that holy Book; as to this hour (1593), praised be the Lord, I have been doing at all such times as conveniently I might have occasion."

Here is a trait seldom surpassed in the history of boyhood. The mind of his great contemporary, Galileo, when a few years older, was also roused to powerful activity in the house of God. But his eye, not his ear, was attracted; a characteristic difference between the practical and the speculative philosopher, which continued throughout their respective careers. In the cathedral of Pisa, to which city the young Italian had been sent for education at the university, he fixed his gaze upon the vibrations of a lamp. Amid the pageantry of that worship against which Napier warred, and of which Galileo was destined to be a victim, he watched the isochronal movements of the chain, and measured them by the beatings of his pulse. The result was the pendulum.

The *scientific* fruits of Napier's attention to the Protestant divine's sermon, if less direct, were no less valuable than those of Galileo's inattention to the papal service. It was during this dreary adventure, undertaken in his very boyhood, that the genius of the Scotch mathematician found its proper elevation. No sooner had he determined to "make plain" the mysteries of St. John, than he found himself constrained to grapple, nothing loth, with the difficulties of numerical science. Doubtless, "he lisped in *numbers*, for the *numbers* came." Intuitively his work assumed a mathematical form. "Being of purpose," he says, "to expound and open up the mysteries of this revelation by a two-fold discourse, the one *paraphrastical*, the other *historical*, both confronted together, I have thought good to premit, by way of introduction, a reasoning for the investigation of the true sense and meaning of every notable mystery thereof, and to set the same in form of *propositions*, as near the *analytic* or *demonstrative* manner as the phrase and nature of holy scriptures will permit." Then, after elaborately, but lucidly, disclosing his *modus operandi*, he proceeds to the groundwork of his exposition, his first object being to demonstrate the meaning of "dates and chief reckonings hid under terms." Such studies could not fail to direct the natural tendency of his mind to numerical calculations. He had to extricate and determine a system of chronology ; to reckon dates, and the number of days, weeks, and years ; and to resolve the problem of "a time, times, and half a time." In the progress of this undertaking, his natural genius inevitably impelled him to the attempt of ascertaining, by interpretation and calculation the precise time, or near advent, of the end of all things. His theological calculations led him, or rather misled him, to the conclusion (by no means dogmatically expressed) that "the day of judgment *appears to fall* betwixt the years of Christ 1688 and 1700." We now know that the awful period only brought us King William. The cautious Sir Isaac Newton was wise in not repeating that daring attempt of his great prototype. But in thus vainly seeking for the day of judgment, Napier kept calculating and calculating, till he found the LOGARITHMS.

The idea of the near approach of the latter days has been so prevalent in every age, including that of the Apostles themselves, and is so inevitable to those who study the subject deeply, that to infer from it a weak or unsound state of mind, is greater weakness in itself. It is from the individual's mode of arriving at and treating such conclusion, that weakness or wildness is to be discovered. Napier's reasoning is scarcely to be impugned. He refers to the text, "But of *that day*, and *that hour* knoweth no man, no not the angels which are in Heaven; neither the Son, but the Father." His argument, however, from the same chapter, is not easy to meet, that Christ's knowledge yields to that of the Father only in respect of the precise day and hour; and that the Son was even careful to instruct his disciples that they might know the signs. He compares this to our knowledge of the approach of death; and he adds, "To what effect were the prophecies of Daniel, and of the Revelations, given to the Church of God, and *so many dates* of years, and circumstances of time foreshowing the latter day, contained therein, if God had appointed the same to be never known or understood before that day come?" He also quotes Daniel:—"*Signa librum ad tempus statutum; multi pertransibunt et multiplex erit scientia:* Seal the Book till the appointed time; many shall go to and fro, and knowledge shall be increased."

Surely Sir Isaac Newton must have seen one at least of the many editions, in various languages, of Napier's work. How little does the English philosopher, writing a century after, differ on this momentous subject from his great predecessor. "In the very end," says Newton, "the prophecy shall be so far interpreted as to convince many. 'Then,' saith Daniel, 'many shall run to and fro, and knowledge shall be increased.' For the Gospel must be preached in all nations before the great tribulation, &c.: But if the last age, the age of opening these things, be *now* approaching, *as by the great successes of late interpreters it seems to be*, we have more encouragement than ever to look into these things: if the general preaching of the Gospel be approaching, it is to us and to our posterity that those words mainly belong,—'In the time of the end, the wise shall understand, but none of the wicked shall understand.'"

It will be obvious to any one who compares their writings, that Napier, even upon the problem of the last day, is no more wild and visionary than Newton was. The former, who, be it remembered, belonged to a very different age, is a little more precise and courageous in his examination of this mystery of mysteries, and even commits himself by hazarding a computation of the period. But Sir Isaac Newton, with the immense advantage of a century's additional light and experience, and with the commentaries of Mede between him and Napier, also hazards a conjecture of the end of all things being at hand, wields the very argument of Napier, and quotes the same texts to prove it. The difference between them is merely this, that Napier, upon comparing *his* chronology of the world with the signs of his times, supposed that the period of "understanding" by the wise had arrived; while Newton only gathered from his comparison that "the age of opening these things" was approaching. Accordingly, the one perilled a calculation; the other said, there was "*encouragement* to look into these things." And both laboured in vain; as the conflicting commentaries of a Keith, a Cumming, and an Elliott in our own times, may suffice to prove.

III. Coincidence of their Treatises on Arithmetic and Algebra.

But while Napier laid the feeble hand of mortality upon the Apocalypse, he clutched the inchoate system of NUMBERS with the grasp of a giant. He set himself to develope that wing of applicate science, with the same systematic energy with which he had endeavoured to unveil Daniel and Saint John. This is manifest from what he tells us himself. In the first place, he says, he had long and laboriously wrought out,—"*à me longo tempore elaboratum*,"—his Canon of the Logarithms, prior to their publication in 1614. There is evidence that he had mastered the invention before the year 1594. Then he excogitated (*excogitavimus*) the mechanical system of figured rods, called *Rhabdologia*, or "Neper's Bones," for the benefit of those who might be distrustful of the artificial system of his Logarithms. His "Promptuary of Multiplication," he states to be the latest of all his inventions,—"*omnium ultimo à nobis inventum sit hoc multiplicationis promptuarium*." He had previously invented his mode of calculating with the *Abacus*, or chess-board; in the preface to which he again refers to the origin of all these valuable inventions; namely, that he had made it the labour of his life to rend the fetters with which applicate science was still clogged and retarded. He says that he devoted "every moment of my leisure,"—doubtless, from what he considered his chief calling, the Revelations,—"to the invention of these compendious methods of calculation, and to the inquiry, by what means the labour and toil of calculation might be removed."

The great mass of Napier's loose papers and scientific manuscripts, along with a Bible containing his autograph, were deposited in a chest, placed in the garret of a country house belonging to the representative of that younger son, Robert Napier, whose precious illustrations of the divine art of Hermes we have to a certain extent ventured to disclose. Robert was his father's literary executor, and published his posthumous work, the mode of *constructing* Logarithms. Some time in the last century, the domestic calamity of destruction by fire of that country house, assumed somewhat of a public one by reducing to ashes the chestful ot scientific manuscripts. These had never been explored, but their value, at least to the history of science, may be surmised from some remnants very accidentally saved. There had been previously transmitted to the then Lord Napier, as head of the house, two manuscripts, considered curious specimens; the one being that treatise of Alchemy, by Robert Napier himself; and the other, "The Baron of Merchiston, his Book of Arithmetic and Algebra." The contents of what perished, no man alive can tell. That which has been saved is a Latin treatise, "De Arte Logistica," comprehending both Arithmetic and Algebra.

It forms another of the many curious coincidences between the genius of Napier and of Newton, that the latter also wrote a Latin treatise of Arithmetic and Algebra, entitled, "Arithmetica Universalis," being the substance of his lectures at Cambridge. It is very interesting to compare Newton's work with Napier's, which for a whole century before had been concealed in a Scotch charter-chest. In this discourse I can only afford a single example; namely, their respective elementary introduction to the refined and subtle philosophy of *plus* and *minus*. A literal translation from each will be excused.

"Quantities," says Sir Isaac, "are either *affirmative*, or greater than nothing (*majores nihilo*), or *negative*, or less than nothing (*nihilo minores*). So in human affairs *possessions* may be called affirmative goods; *debts*, negative goods. And in locomotion, progression may be called affirmative motion; retrogression, negative motion; the first being an *increase*, and the other a *decrease*, of the path commenced. Negative quantities are indicated by the sign —, affirmative have the sign + prefixed."

Bishop Horsley, Newton's very learned editor, commenting upon this passage ("Opera," t. i., p. 3), says :—

"If I mistake not, Albert Girard, doubtless a consummate mathematician, was the very first (*omnium primus*), to use the expression *nihilo minores*, by a rude figure of speech utterly unknown to Diophantus and Vieta, and which I wish Descartes, and some of our mathematicians, had not so eagerly adopted."

Dr. Hutton, in his "History of Algebra," thus follows what we shall immediately show to be a complete, and, in such hands, a strange mistake :—

"Girard was the first who gave the whimsical name of *quantities less than nothing*, to the negative ones."

And what is yet more remarkable, the great Scotch professors, Leslie and Playfair, fell into the same blunder as to the origin of the phrase, the one condemning, the other defending a nomenclature adopted by Newton :—

"Girard," says Leslie, "was possessed of fancy as well as invention ; and his fondness for philological speculations led him to frame *new terms*, and to adopt certain modes of expression which are not always strictly logical ; though he stated well the contrast of the signs *plus* and *minus;* he *first introduced* the very inaccurate phrases of *greater* and *less than nothing.*"

Then Playfair says :—

"*Girard is the author* of the figurative expression which gives the negative quantities the name of *quantities less than nothing;* a phrase that has been severely censured by those who forget that there are correct ideas which correct language can hardly be made to express."

Albert Girard was a Flemish mathematician, who flourished after the time of Napier. His "Invention Nouvelle en Algèbre," was not printed until 1629. Napier's "Canon of Logarithms" was first published in 1614, just three years before his death. Whatever Horsley and Hutton might have done, we are certain that Leslie and Playfair, whose admiration of the genius of Napier was unbounded, would have blushed to have had it pointed out to them, from a work worthy of being placed beside Newton's *Principia*, and which they ought to have known by heart, the very nomenclature they all so pointedly ascribe to Girard. At the outset of Napier's published Canon, we find the most precious practical application of that doctrine of *plus* and *minus* ; which also forms a valuable chapter of his *unpublished* manuscript treatise on Logistic, where it is expounded in terms exactly similar to what we have quoted before, from Newton's "Universal Arithmetic," but more fully and systematically. Had the proof rested upon Napier's manuscripts, only brought to light by myself at no distant time ago, their mistake would have been natural. But how came these four mathematical *savants* to ignore this important passage in that great work, the "Canon Mirificus Logarithmorum?" We give it from a translation published in 1616, and revised by Napier himself:—

"Therefore we call the logarithms of the sines *abounding*, because they are always greater than nothing (*majores nihilo*), and set this mark before them, +, or else none; but the logarithms which are less than nothing (*minores nihilo*), we call *defective* or wanting, setting this mark, —, before them."

This contradicts Horsley, Hutton, Playfair, and Leslie; and the contradiction is derived from a work of the greatest interest and importance to Science next to the "Principia Mathematica." Napier's mode of demonstrating the Logarithms, as we shall have occasion presently to notice more particularly, was by the idea of locomotion, namely, the motion of two points; one he conceived to generate a line by *increase*, in equal proportions in equal moments; the other, to facilitate his operations, he conceived in the *decreasing* ratio, namely, a moving point cutting off small parts continually, each small part bearing the same relative proportion to the line from which it was cut off. This, in fact, is an exemplification of the doctrine of *plus* and *minus* the very same as that which we have already quoted from Sir Isaac Newton's explanation of what he termed *affirmative* and *negative* quantities. Napier, by a phraseology less liable to cavil, had called them *abounding* or *abundant*, and *deficient* or *defective* quantities. Now, it is in his manuscript "De Arte Logistica," and before evolving the admirable expedient of Logarithms, that the Scotch mathematician, a hundred years before Newton, laid the groundwork for his future logarithmic demonstrations, in his beautiful general treatment of the subject of *plus* and *minus*; and we may here translate a passage from Napier's chapter, "De quantitatibus Abundantibus et Defectivis," for comparison with the literal translation already given from the work of Newton.

"Abundant (*abundantes*) quantities are those which are greater than nothing (*majores nihilo*) and carry the idea of *increase* along with them. These have either no symbol prefixed, or this one, +, which is the copulative (*copula*) of increase. Thus, if you are not in debt, and your wealth be estimated at 100 crowns, these may either be noted 100 crowns, or + 100 crowns ; and are to be read, a hundred crowns of increase ; always signifying wealth and gain. Defective (*defectivæ*) quantities are those which are less than nothing (*minores nihilo*) and carry the idea of diminution along with them. These are always preceded by this symbol, —, which is the copulative of diminution. Thus, in the estimation of his wealth whose debts exceed his goods by 100 crowns, justly his funds are thus pre-noted, — 100 crowns, and are to be read, a hundred crowns of *decrease;* signifying always loss and defect. I have already shown that defective quantities have their origin in subtracting the greater from the less."

He then proceeds to lay down the general rules of the arithmetic of *plus* and *minus*, and to connect the chapter with the rest of his system, in a manner certainly not surpassed by Newton, Maclaurin, and Euler, in a far riper age. At the same time he was perfectly well aware that he was dealing with a most fructiferous department of his subject. In a subsequent chapter, of great interest and curiosity, when explaining a most original device of his own for a new symbolical notation of irrational roots (at a time when the modern algebraic notation was unknown), he refers to his chapter of *plus* and *minus* in these words :—

"Seeing, therefore, that a *surd uninome* may be the root either of an abounding or of a defective number, and that its index (*index*) may be either even or odd, from this *fourfold* cause it follows, that some surds are abounding, some defective, some both abounding and defective, which I term *gemina*,—some neither abounding nor defective, which I term *nugacia*. The foundation of this *great algebraic secret,* I have already laid in the sixth chapter of the first book ; and though hitherto *unrevealed by any one else, so far as I know,* the value of it to this art, and to mathematics in general, shall presently be made manifest."

The internal evidence is quite conclusive, that this is no allusion to his great discovery of the Logarithms which had not yet occurred to him. He used the word *uninome* (*uninomium*), to signify a simple uncompounded "*concrete* number proper," which he defines to be "the *root* of an *irreducible* number, and these roots are commonly called *surd* and *irrational.*" Compounded quantities of this kind he called *plurinomia.*

Napier had too strong a hold of his subject to reject these latent and ineffable roots as *no quantity at all.* He views them, indeed, in their proper *concrete* character of quantity or magnitude, rather than a *discrete* number or multitude; and he calls them "nomina," because susceptible, he says, "rather of being *named* than *numbered*." But he considered these quantities so profoundly, as to discover all their computative properties, and fully to illustrate them under the operation of all the rules of Arithmetic relative to *discrete* number and quantity.

There is also a mathematical quantity which has obtained the startling designation of *imaginary* or *impossible quantities*. Playfair, speaking of Girard, in the passage already referred to regarding quantities less than nothing, says, "the same mathematician conceived the notion of *imaginary* roots." This accomplished professor was not aware of the existence of Napier's manuscript. There can be no doubt, that by "nugacia," the old Baron means the *impossible* quantity; and his manuscript proves that he was the first to conceive the idea, and to propound its use, in the Arithmetic of Surds, and Theory of Equations. He explains minutely the nature of such quantities, invents a notation for them, and, with the consciousness of algebraic knowledge and genius, fears not to describe it as "a quantity absurd and *impossible*, nonsensical, and signifying nothing." He was, in fact, the first inventor (unknown to the world, his manuscript remaining unfinished and unpublished) of the Arithmetic of Surds, hitherto assigned to Girard. Early and rude as was the period of algebraic science to which we must refer Napier's manuscript, we find him treating these mysterious quantities with the most perfect command of their mathematical qualities, and looking forward with confidence and exultation to his own future applications of this "great algebraic secret." Nothing can be more interesting to the mathematical student than his opening chapters of Equations. They prove that he was among the very first thoroughly to understand that redoubtable department of Numbers, his treatment of which will stand a comparison with the best works of his illustrious successors in that walk, from Harriot to Euler. Upon the strength of this manuscript, then (edited by me for the Bannatyne and Maitland Clubs in 1839), I claim for Napier the invention of the Arithmetic of Surds, the application of which to a higher department of Algebra is the secret to which he alludes in the passage already quoted.

IV. Coincidence of the Binomial Theorem.

Even Euler's chapter of the Binomial Theorem, the algebraic glory of Newton, and engraved on his tomb, presents another remarkable coincidence between Napier and Newton. Euler, in his Algebra, presents the student with a table of integer numbers, arranged in a triangular form, from which he discovers the law whereby binomial coefficients are formed. From this table, indeed, he proceeds to deduce the Binomial Theorem itself, and concludes his chapter with these words:—

"This elegant theorem, for the involution of a compound quantity of two terms, evidently includes all powers whatever; and we shall afterwards show how the same may be applied to the extraction of roots."

Now, the 7th chapter of the 2nd book of Napier's manuscript is entitled, "Of finding the rules for the extraction of roots;" and therein occurs a triangular disposition of integer numbers, precisely the same as that displayed by Euler. In the manuscript, however, the numerals are inclosed in a diagram of small hexagonals, forming a figure of singular beauty, for drawing which precise directions are given, and the inventor adds, "and thus you have my triangular table, filled with little hexagonal areas." Manifestly the old Scotch philosopher required no more than the Cartesian notation, to have given the Binomial Theorem itself; a fact I proceed to illustrate.

In more modern times, the celebrated Blaise Pascal, one of the most profound minds ever created, has obtained the very highest praise for his "Arithmetical Triangle." It is just Napier's table (of which Pascal knew nothing) in a far less beautiful diagram. Montucla, in his "History of Mathematics," says of it,—"Les usages de ce triangle arithmétique sont nombreux, et c'est une invention *vraiment original,* et *singulièrement ingénieuse.*" Nay, so intimately connected with the Binomial Theorem are the properties of this triangle of whole numbers, that Bernoulli claims for Pascal the famous theorem itself. In his annotations on a work of Mr. Stone's upon the infinitesimal analysis, where the latter speaks of that "marvellous theorem," Bernoulli notes,—"Pour l'élévation d'un binome à une puissance quelconque: *Nous avons* trouvé ce merveilleux théorème *aussi-bien* que M. Newton, d'une maniere plus simple que la sienne : Feu M. Pascal *a ete le premier* qui l'a inventee."

Bernoulli was mistaken. Without pretending to enter into the question which that great mathematician so promptly determines to his own satisfaction, this much we may say, that Pascal, in his discovery of that triangular configuration of integer numbers, and its important properties, was not "le premier qui l'a inventée." It lay hidden, long before his time, amid the dusty records of an ancient Scottish charter-chest, and is minutely and profoundly expounded by Napier, in the progress of a complete digest of the whole art of Logistic. (*See Appendix to this Paper.*)

V. Coincidence in Fluxions.

"Newton's Fluxions," that refined expansion of the principle of the Logarithms which opened a new era in the science of calculation, are terms scarcely less familiar to those who do not understand them, than to those who do. But where did he get the term *Fluxions?* The reserved Sir Isaac was not in the habit of pausing to record the external suggestions and impulses which directed him to his rapid triumphs. When his unfortunate controversy with Leibnitz constrained him to give some account of his discovery of Fluxions, he so expressed himself as to seem to say, that the geometrical mode of flowing quantities, whereby he demonstrated the new calculus, and the relative terms *fluxions* and *fluents,* were original ideas, arising spontaneously in his own mind. That Newton ever meant to conceal any derivative impulse, or the source of any aid which his own preeminent genius had ever derived from a gifted predecessor, is not to be imagined ; and one might as well accuse the sun of being a plagiarist of light, as the author of Fluxions of plagiary in mathematics. Nevertheless, the following passage, which we translate from the Latin of Sir Isaac's "Introductio ad Quadraturam Curvarum," ("Opera,"t. i., p. 333), is somewhat too exclusively expressed :—

"I here consider," he says, "mathematical quantities, not as consisting of infinitely small parts, but as described by a continued motion. Lines are described, and therefore generated, not by the apposition of parts, but by the *continued motion of points,*" &c. "Therefore, considering that quantities which increase in equal times, and by increasing are generated, become greater or less according to the greater or less velocity with which they increase and are generated, I sought a method of determining quantities from the velocities of the motion, or *increments*, with which they are generated; and calling these velocities of the motions, or increments, *fluxions*, and the generated quantities *fluents*, I fell *by degrees*, in the years 1665 and 1666, upon the method of Fluxions, which I have made use of here in the Quadrature of Curves."

That distinguished Scotch mathematician of the last century, Colin Maclaurin, the friend and assiduous commentator and expounder of Newton, in like manner tells us, how "Sir Isaac Newton considered magnitudes as generated by a flux or motion, and showed how the velocities of the generating motions were to be compared together;" and then he adds,—"The *method of demonstration*, which was *invented by the author of Fluxions*, is accurate and elegant; but we propose to begin with one that is somewhat different," &c. ("Treatise of Fluxions," Vol. i, pp. 2, 3.) And even Professor Leslie, a vast admirer of Napier's, following probably the same lead, entirely ignores the Scotch mathematician when thus recording the Calculus of Newton:—

"The notion of flowing quantities, *first proposed by Newton*, and from which *he framed* the terms *fluxions* and *fluents*, appears on the whole very clear and satisfactory; nor should the metaphysical objections of introducing ideas of motion into Geometry have much weight: Maclaurin was induced, however, by such cavilling, to devote half a volume to an able but superfluous discussion of this question."

This statement, from such a quarter, might have caused the old Scottish Baron to rise from his grave and exclaim:—

"Me, me, adsum qui feci, in me convertite ferrum."

It is the more remarkable, that Sir John Leslie was a most accomplished and ingenious explorer of the antiquities both of Science and of History. Why speak merely of Colin Maclaurin having superfluously defended Newton from the metaphysical cavil of introducing ideas of motion into Geometry, and why did Maclaurin himself break that lance as if the quarrel were a new one, seeing that, a century before, the inventor of Logarithms had been canvassed, criticised, and chided by the great mathematicians of Upper Germany, for introducing that very same idea of motion into Geometry, and was publicly and enthusiastically defended against them all by the immortal Kepler, long before Newton was born?

"When," says Kepler, "in the year 1621, I passed into Upper Germany, and debated everywhere with those skilled in the mathematical sciences, concerning the Logarithms of Napier, I discovered that they, of whose minds years had diminished the quickness in proportion to the acquisition of caution, were loth to admit this kind of numbers in place of the usual canon of sines. They said it was derogatory to a professor of mathematics to exhibit such childish exultation about any compendious method of calculation, and at once to receive into practice, without even a legitimate demonstration, a form of calculus which might some day, and when least expected, involve the unwary in the snares of error. They complained that this *demonstration of Napier's* depended upon the fiction of some kind of geometrical motion, whose lubricity and *fluxibility* (*lubricitas et fluxibilitas*), was quite inept to sustain the severe march of geometrical reasoning and demonstration."—*Kepleri Chilias*, &c., p. 113, 1624.

And so this immortal genius proceeded, with great enthusiasm, and most amusing indignation, to defend the old Scottish Baron (of whom he had never heard until the Logarithms appeared), and to illustrate Napier's Fluxions, just as Maclaurin did by Newton's a century later.

Nor was this idea of geometrical motion, as a means of demonstrating new powers of calculation, either latent or barren in the hands of Napier. He announced it to the world at the very threshold of a work destined immediately to create a great revolution in science, abstract and applicate. The discovery of Logarithms does not afford an instance of the rough pebbles which Newton was destined to cut and polish. The original Canon, the most unaided and unsuggested of inventions, was presented to the world complete in all its parts, ready for the work of a new era in calculation, "*in seipso totus teres atque rotundus.*"

In the first page, first chapter, and first definition, in Napier's first work published in 1614, these words occur :—

"Linea æqualiter crescere dicitur, quum *punctus eam describens* æqualibus *momentis* per æqualia intervalla progreditur."

Of this the author himself revised a translation, wherein that passage runs thus :—

"A line is said to increase equally, when the point describing the same goeth forward equal spaces in equal times, or *moments.*"

And in proceeding with his demonstration he thus expresses himself :—

"Sit punctus A, à quo ducenda sit linea *fluxu* alterius puncti, qui sit B ; *fluat* ergo primo *momento* B ab A in C," &c.

And then follows the corollary :—

"Unde hoc *incremento* quantitates æqui-differentes temporibus æqui-differentibus produci est necesse."

Now, in the first place be it observed, that Leibnitz, in the "Acta Eruditorum" for January, 1705 (p. 34), which commenced his controversy with Newton, uses the very language of Napier. Speaking of his great rival's introduction to his Quadrature of Curves (already quoted), he says:—

> "That it may be better understood, be it known, when any quantity increases continuously, as a line, for example, increases by the flowing of a point describing the same (*fluxu puncti quod eam describit*), those momentary increments (*incrementa illa momentanea*) are to be called differences (*differentias*); namely, the difference between the original quantity, and that which is produced by the momentary motion; and hence the *differential* calculus."

Leibnitz then goes on to state, that Newton had called the very same thing *Fluxions.*

But, in the next place, even in the passages we have quoted from Napier's Canon, so far as the fundamental principle, and the exact nomenclature are concerned, may be perceived more than the coming shadows both of the *fluxions* or flowing quantities of Newton, and of the *differentiæ*, or differential calculus of Leibnitz. And surely it is worthy of remark, that the very words which the whole of Newton's commentators and biographers refer solely to him, without an allusion to the demonstration of the logarithmic fluxions, all find their exact Latin equivalents at the very outset of Napier's Canon, namely, *incrementum, decrementum, momentum, fluxu,* and *fluat;* while, a little further on, there repeatedly occurs the term adopted by Leibnitz, namely, *differentiæ.*

The genesis of a line by the motion of a point (the most simple idea in nature) is indeed a geometrical notion at least as old as Archimedes. But with the Greeks it was, comparatively, a barren idea, and produced nothing in mathematics. It produced neither the Fluxions of Napier, nor of Newton, nor of Leibnitz. The very first great fruit of that geometrical idea was the Logarithms; and as for the nomenclature, we know of no earlier use of the term *fluxions,* than Napier's *fluxu* and *fluat.*

Neither is this a mere unimportant coincidence of phrases. So strong is the mathematical affinity, in this matter of Fluxions, between Napier and Newton, that when Maclaurin applied his most ingenious mind to expound Newton's fluxionary method, he wrote a chapter "Of the grounds of this method," which serves equally well to illustrate Napier's Logarithms or Newton's Fluxions. And even Dr. Hutton, who in some respects has done great injustice to Napier, in his elementary history of the Logarithms, finds himself constrained to observe :—

"Napier's manner of conceiving the generation of the lines of the natural numbers, and their logarithms, by the motion of a point, is *very similar* to the manner in which Newton *afterwards* considered the generation of magnitudes in his doctrine of Fluxions : and it is also remarkable, that in article second of the 'Habitudines Logarithmorum, et suorum naturalium numerorum invicem,' in the appendix to the 'Constructio Logarithmorum,' Napier speaks of the *velocity of the increments,* or *decrements,* of the logarithms, in the *same way that Newton does,* namely, of his *Fluxions ;* where he shows that those velocities, or fluxions, are inversely as the sines, or natural numbers, of the logarithms ; which is a necessary consequence of the *nature of the generation,*" &c.

And Dr. Hutton mentions this more particularly afterwards, when he says :—

"I shall here set down one more of these relations, as the manner in which it is expressed (by Napier) is *exactly similar* to that of *fluxions* and *fluents ;* and it is this : Of any two numbers,—'As the greater is to the less, so is the *velocity* of the increment, or decrement (*incrementi aut decrementi*) of the logarithms at the less, to the velocity of the increment or decrement of the logarithms at the greater :' That is, in our *modern notation,* as X : Y : : y : x ; where x and y are the *fluxions* of the logarithms X and Y."

Here the mathematical language of Napier, when expounding his Logarithms, is shown to be *identical* with that of Newton, when expounding (in *Cartesian notation*) his method of Fluxions ; and to this illustration must be added the very significant fact already pointed to, that Newton's term *fluxions,* is also to be found,—and found so far as I know for the first time,—in Napier's *fluxu* and *fluat.*

It was necessary to be thus particular, as the assertion may have startled some of my audience, that, while the ancient Scotch philosopher was dabbling, and dreaming, and doubting in alchemy (just as the English philosopher was doing a century later at Cambridge, even when he had made conquest of the fluxionary calculus), the fundamental principle, and the very nomenclature of that immense impulse to calculation, *Fluxions,* were already conceived and recorded by Napier, in works which for their great mental power, and universal practical application, well deserve to be placed beside the "Principia Mathematica." Nor is it too much to say, that the illustrious Newton's exposition of his Genesis of Fluxions in that controversial preface to his "Quadrature of Curves," would have been more perfect and valuable, as a page of the history of science, if he had condescended to add:—

"And after this manner Napier of Merchiston in Scotland, a century before my time, by drawing a moveable point along a right line, taught the Genesis of Logarithms which have become so indissolubly interwoven with the fluxionary calculus; and when I speak of quantities becoming greater or less according to the greater or less velocity with which the increase and decrease are generated; and of determining quantities from the *velocities* of the motions or *increments* with which they are generated; and when I call these *velocities* of the motions or increments, *Fluxions,*—I avail myself of Napier's demonstration; I adopt his mathematical reasoning; I use his very expressions, *fluxu* and *fluat,* and *incrementi* aut *decrementi;* and, with the aid of the Cartesian *notation,* I repeat, and expand into the regions of a new and more powerful calculus, this his own original proposition,—'Ut sinus major ad minorem, ita *velocitas incrementi,* aut *decrementi,* logarithmorum apud minorem ad velocitatem incrementi aut decrementi logarithmorum apud majorem.'"

Had this been the language of Newton, how valuable would it have been to the fame of Napier, *and how true!*

The *greatest lever of Newton's fame was the Logarithms.* Not only was their practical aid indispensable to his calculations, but their mathematical principle, and most suggestive properties, are intimately connected with his algebraic operations in their more transcendental departments. Newton could not fail to know this; and, had the question been put to himself, surely he would not have failed to acknowledge it. Yet throughout his voluminous collected works I have been unable to discover a single allusion to Napier, or to his great invention. Newton commenced his ascent to the pinnacle of his fame, the throne of mathematics, having the beautiful system which has obtained the name of the *Arabic* notation, complete to his hand. Logistic was then ready for its gradual expansion through the new algebraic notation into the higher calculus. But Napier *had to complete* the Indian, or Arabic system of arithmetic, which he found inchoate and undeveloped in the XVIth century. Wallis, whose algebraic works were the earliest impulsive studies of Newton, tells us, while tracing the history of Algebra,—

"There are two improvements, very considerable, which *we have added* to the algorism of Arithmetic since we received it from the Arabs; to wit, that of Decimal Fractions, and that of Logarithms."—*Treatise of Algebra*, p. 15.

But who added them? When Napier attacked the mysteries of Numbers, neither improvement existed. When death unexpectedly cut short his labours, at the untimely age of sixty-seven, to himself belonged the chief merit of the one, and the sole glory of the other. Indeed his geometrical idea of motion, which he took to generate *ratios*, or proportions,—or as *Delambre* acutely remarks, when doing all honour to Napier in his "History of Astronomy," "Cette idée de *fluxions*, et de *fluentes* qu'on a *depuis* reprochée à Newton,"—was analogous to the law of the Arabic notation, where the significant digit may be conceived to generate an infinite decuple progression, by travelling in a line by equal steps from right to left. But it was Napier himself who *completed* the plan. The working of *decimal fractions* is just the infinite *decreasing* progression from unit in the opposite direction. When this principle is thoroughly understood, it is simply to be operated upon by placing a point between. The system had been previously mooted on the continent in a ruder form, indicating a less ripe consideration and stage of the system. But, says Professor Leslie,—

"It was our illustrious countryman Napier that brought the notation of decimals to its ultimate simplicity, having proposed in his 'Rhabdologia' to reject entirely the marks placed over the fractions, and merely to set a point at the end of the units. But his sublime invention of Logarithms about this epoch eclipsed every minor improvement, and as far transcended the *denary* notation, as that had surpassed the numeral system of the Greeks."—*Dissertation*, p. 587.

When Kepler first turned his mind to the new discovery of Logarithms, he at once pronounced it to be the greatest development which the science of Numbers had received since the introduction of the Arabic notation.

Nothing is more characteristic of the ardent disposition of Kepler, or more consistent with the greatness of his own genius, than the enthusiasm with which he hailed the Logarithms. Writing to a mathematical correspondent at that epoch, after revelling in some of his deepest calculations, he exclaims,—

"But I can conceive nothing more excellent than Napier's method *of proportions* (Logarithms); and yet it is so long ago as 1594, that some Scotchman (*quidem Scotus*), visiting Tycho, even then gave him some hint of the advent of this 'Canon Mirificus Logarithmorum.'"—*Epist. ad Petrum Cugerum.*

This "certain Scotchman" alluded to by Kepler, was Dr. John Craig, an accomplished mathematician, a great friend and correspondent of the Baron of Merchiston. He was also the friend and correspondent of Tycho; and being attached to the household of James VI., as his physician, had accompanied that monarch to Uraniberg, when the great astronomer was honoured with a royal visit. On his return, Craig informed Napier of his adventures there, and that great master of Logistic then caused him to inform the Danish astronomer (for whose imperfect powers of calculation the stars were becoming too many, though he had Kepler for assistant), that he had discovered the Logarithms, and was calculating the Canon. Can a better evidence be afforded of the difficulty and perfect originality of the invention, than the fact, that the Canon remained unpublished for twenty years after this information—admitted by Kepler himself—and neither he nor Tycho fathomed the secret? When it appeared in 1614, Kepler had made some progress with his Rudolphine Tables. Immediately he cast that portion of his labours aside, and recommenced his long expected work upon the basis of the Logarithms. He wrote to Napier that he had done so. Nor was this all. Surely Sir Isaac Newton must have frequently contemplated the ingenious and elaborate device of the engraved frontispiece to those famous Rudolphine Tables, published in 1627. Conspicuous among the tutelary deities elevated round the dome of the Greek Temple of Science there delineated, may be seen a female figure holding in either hand a rod of different proportions, and having the numerals 6931472 arranged in the form of a glory round her head. These numerals compose the hyperbolic, otherwise called the *Naperian* logarithm of half the radius of a circle. It is Kepler's conception of the Genius of the Logarithms. The unequal rods in her hands are symbolical of the fact, that the Scotch philosopher was the first to infuse vitality into the mathematical principle of *ratios* or *proportions;* a principle barren even in the hands of Archimedes, but destined, through Napier, to play a great part in the practical affairs of men, as well as in the accelerated progress of science. Kepler's public monument to his fame, is sufficient consolation for the fact that his own country has erected none, and that Newton forgot to name his benefactor.

VI. Coincidence in Catoptrics and Mechanics.

We have said enough to illustrate our position, that if Newton be the Prince of Mathematicians, Napier is King of Numbers. But his far-searching mind was not satisfied with abstractions destined to develope the long latent powers of calculation. He had not left untouched those inchoate systems of catoptrics and mechanics which the immortal Newton so grandly illustrated. Indeed, his practical value was well understood by the learned among his own contemporaries. Sir John Skene, Lord Clerk Register in the reign of James VI., the great legal antiquary to whom we owe the first collection of our acts of Parliament, the "Regiam Majestatem," the "Quoniam Attachiamenta," and the treatise "De Verborum Significatione," in the course of preparing this last work, came to the word "perticata terræ," which he defines, "from the French word *perche,* much used in the English laws, a rood of land;" and then he adds:—

"But it is necessary that the measurers of land, called *landimers,* in Latin *agrimensores,* observe and keep a just relation betwixt the length and the breadth of the measures which they use in measuring of lands; whereanent I find no mention in the laws and register of this realm, albeit an ordinance thereanent be made by King Edward the First, King of England, the 33rd year of his reign; and because the knowledge of this matter is very necessary in measuring of lands daily used in this realm, I thought good to propose certain questions to John Naper, fear of Merchistoun, a gentleman of singular judgment, especially in the mathematical sciences; the tenor whereof, and his answers made thereto, follows:" &c.

Sir John Skene's treatise was published in 1597; and at the same epoch we find our philosopher in communication with the governments both of his own country and of England, upon the subject of constructing unheard-of instruments of war, for the protection of the whole island from the "enemies of God's truth, and Religion."

Among the papers of Anthony Bacon, preserved in Lambeth Palace, there is a document subscribed, "John

Nepar, fear of Merchiston," in his own hand, and bearing this title :—

"*Anno Domini* 1596, the 7th of June: Secret inventions profitable and necessary in these days for defence of this Island, and withstanding of strangers, enemies to God's truth and Religion."

The subsequent date of receipt by Anthony Bacon is marked by this indorsement :—

"Mr. Stewart: Secretes invention de la guerre, le mois de Juillet 1596."

Colonel William Stewart, Commendator of Pittenweem, sometimes called, "Knight of Houston," was Captain of the Guard to James VI. Archibald Napier, the philosopher's eldest son, was Gentleman of the Bedchamber, and well known to Sir William Stewart. In the year 1595-6, the latter was also entrusted with the important mission of Ambassador Extraordinary to the Emperor of Germany, and other Christian potentates, for the purpose of announcing the King of Scots' alarm at "being informed that the Turk was entered Christendom with a potent army;" and offering his hearty co-operation "to debell the great enemy to our Salviour Christ." ("Hist. of James the Sext."). On the 1st of June, 1596, the famous expedition against Cadiz set sail from England; the land forces being commanded by Essex, and the fleet by the Lord High Admiral Howard. Anthony Bacon (elder brother to the great Verulam) was the devoted friend and secretary of Essex. These facts sufficiently account for the document in question having come from the hands of Sir William Stewart into those of Anthony Bacon, in the month of July 1596. The propositions, indeed, were a day behind the fair; and, probably, having been duly indorsed, were never looked at again. Some days prior to the receipt of it, with no other mirrors than those mirrors of knighthood, Effingham, Essex, and Raleigh,—

"Her Majesty defeated and destroyed the best fleet which the King of Spain had together in any place, and amongst those his ships of greatest fame, and in which all the pride and confidence of the Spaniards were reposed: The captains of them confessed aboard the *Due Repulse*, that forty gallies were not able to encounter one of her Majesty's ships."

This gratifying announcement is quoted from a paper in the Lambeth Collection (Vol. xi., fol. 146), entitled, "The advantages which her Majesty hath gotten by that which hath passed at Cadiz, the 21st of June, 1596."

But, abortive as this patriotic emulation of Archimedes on the part of the ancient Scottish Baron happened to prove, the record remains a most interesting evidence of his own grasp of science in all departments. Manifestly he was no charlatan ; nor would he have made any offer of the kind, unless under the same consciousness of having mastered the inventions, that had prompted, shortly before, his promise of the Logarithms to Tycho, in 1594. It is obvious that in the short *précis* of his inventions which he transmitted to the friend of Essex, Napier intended to conceal rather than expound the particular mode of his Catoptrics, and the principles of the mechanism he had conceived. Like other great inventors, even while benefiting the world by the publication of his Canon of Logarithms, he reserved the secret of his construction of it, which was published by his son after his death. But the document in question affords proof that for years his mind had been occupied with the subject. He positively states that he was now fully prepared, not merely with the *mathematical demonstrations*, but also the practical proof, and *visible demonstrations* of one and all of these warlike instruments, of which he expressly claims to himself *the invention.*

"*First.* The *invention*, *proof*, and *perfect demonstration*, *geometrical* and *algebraical*, of a burning mirror, which, receiving the dispersed beams of the sun, doth reflect the same beams altogether united and concurring precisely in one mathematical point, in which point most necessarily it ingendereth fire : with an evident demonstration of their error who affirm this to be made a parabolic section.

"The use of this invention serveth for burning of the enemy's ships at whatsoever appointed distance.

"*Secondly.* The *invention*, and *sure demonstration*, of another mirror, which receiving the dispersed beams of any material fire or flame, yieldeth also the former effect, and serveth for the like use.

"*Thirdly.* The *invention*, and *visible demonstration*, of a piece of artillery, which, when shot, passeth not lineally through the enemy, destroying only those who stand on the randon thereof, and from them forth flying idly as others do ; but passeth *superficially*, ranging abroad within the whole appointed place, and not departing forth of the place till it hath executed its whole strength, by destroying those that be within the bounds of the said place.

"The use thereof not only serveth greatly against the army of the enemy on land, but also by sea it serveth to destroy, and cut down, and unshot the whole masts and tackling of so many ships as be within the appointed bounds, as well abroad as in large, so long as any strength at all remaineth.

Fourthly. The *invention of a round chariot of metal*, made of the proof of double musket, whose motion shall be by those that be within the same, more easy, more light, and more speedy by much than so many armed men would be otherways.

"The use hereof, in moving, serveth to break the array of the enemy's battle, and to make passage; as also, in staying and abiding within the enemy's battle, it serveth to destroy the environed enemy by continued charge of harquebuss through small holes; the enemy meantime being abashed, and altogether uncertain what defence or pursuit to use against a *moving mouth of metal.*

"These inventions, besides *devices of sailing under the water*, with divers *other* devices and stratagems for harming of the enemy, by the grace of God, and work of expert craftsmen, I hope to perform.

"Jo. Nepar, fear of Merchistoun."

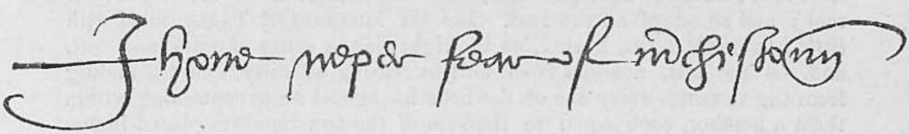
A hasty reading of our philosopher's first proposition might lead to the idea that he had fallen into the mistake of denying the well-established proposition, that a parabolic speculum reflects the solar rays to a burning point, the focus of the parabola. The hint lies deeper, and is very interesting. He proposed to burn "the enemy's ships at *whatsoever* appointed distance." But how could a parabolic speculum be constructed of such dimensions, that its focus, or burning point, could be thrown to any distance? And hence the famous exploit recorded of Archimedes came to be regarded as a fable; because many vain attempts had been made to realize it; all founded, however, upon a law of catoptrics undeniable in the abstract, and practicable within certain limits. To exceed this limit, indeed to be independent of any particular limit, was the object of the Scottish Archimedes, and one which he professed to have accomplished. Centuries after his time, we find the question keenly discussed by the *savants* of science. Montucla sets himself to controvert the idea of Archimedes's experiment with parabolic burning glasses having succeeded upon the distant ships: "En vain," he says, in his "History of Mathematics," t. i. p. 232, "proposeroit on, avec quelques-uns, une combinaison de miroirs paraboliques, à l'aide de laquelle ils ont prétendu produire un foyer continu dans l'étendue d'une ligne d'une grande longueur; ce n'est-la qu'une idée mal refléchie, et dont l'exécution est impraticable, par bien des raisons."

Napier, nearly two centuries before, knew that as well as Montucla and Buffon. He expressly proposes to demonstrate the impossibility of the parabolic curve being so applied with effect beyond a very limited distance ; but at the same time he was prepared, he says, with—"Proof, and perfect demonstration, geometrical and algebraical," of his own invention, upon some other principle of catoptrics "for burning the enemy's ships at *whatsoever appointed distance.*" The inventor of Logarithms was the last man in the world to have thus promised proof and perfect demonstration of the kind loosely or crudely, or upon grounds of which he did not himself feel perfectly sure. The great Descartes, no doubt, issued his fiat—but all reasoned upon the arguments of the very limited range of the parabolic focus,—that—

"Hence is is obvious, that, from a crude conception of optics, impossibilities have been imagined ; and that those famous burning mirrors of Archimedes, by which he is said to have consumed a fleet in the distance, must either have been mighty big, or, what is more probable, are a fabulous creation."—*Dioptrices*, c. viii. p. 22.

Napier's reply obviously would have been,—"But Archimedes knew better than to make such an attempt with a parabolic speculum ; I will show you how he did it, upon a different principle." Accordingly, what do we find in the second century after the Scotch philosopher's announcement? The Count de Buffon practically controverting the dogma of Descartes, by operating with a congeries of plain mirrors, and setting fire to planks of wood, in less than a minute and a half, at the distance of 150 feet, and also at 210 feet in a like time. And this principle he showed to be capable of an extension only controlled by the limits of the materials, and as Napier said, "the necessity of the aid of expert craftsmen." We refer to Buffon's "Invention de Miroirs pour brûler à de grandes distances," in the supplement to his Natural History, i. 399.

Napier's second invention, which to us seems as fanciful as the attempt to read the sun-dial with the light of a candle, namely, to operate in like manner with "any material fire or flame," I do not profess to illustrate ; but that he was very far in advance of his age, and of all the conceptions of the most scientific war-providers of the XVIth century, and was, nevertheless, indulging in no fanciful speculations, but had anticipated the most dreadful yet now common engines of modern warfare—let the Congreve rockets, the diabolical shells, spherical case shot, and other such awful devices, which characterised the "infernal fire" at Sevastopol, bear witness. And if we allow that *steam* can realise his fourth invention—that "moving mouth of metal," the motion easily and speedily directed by those *within* "a round chariot of metal made of the proof of double musket," it must be conceded that Napier was no less practical in his science than Newton, and that the old Scottish Baron now stands fully justified in those neglected proposals which he transmitted to Anthony Bacon in the year 1596.



APPENDIX.

The subject of our fourth coincidence, relative to the Binomial Theorem, is so curious and interesting, that a more particular illustration will be acceptable to the student of mathematics.

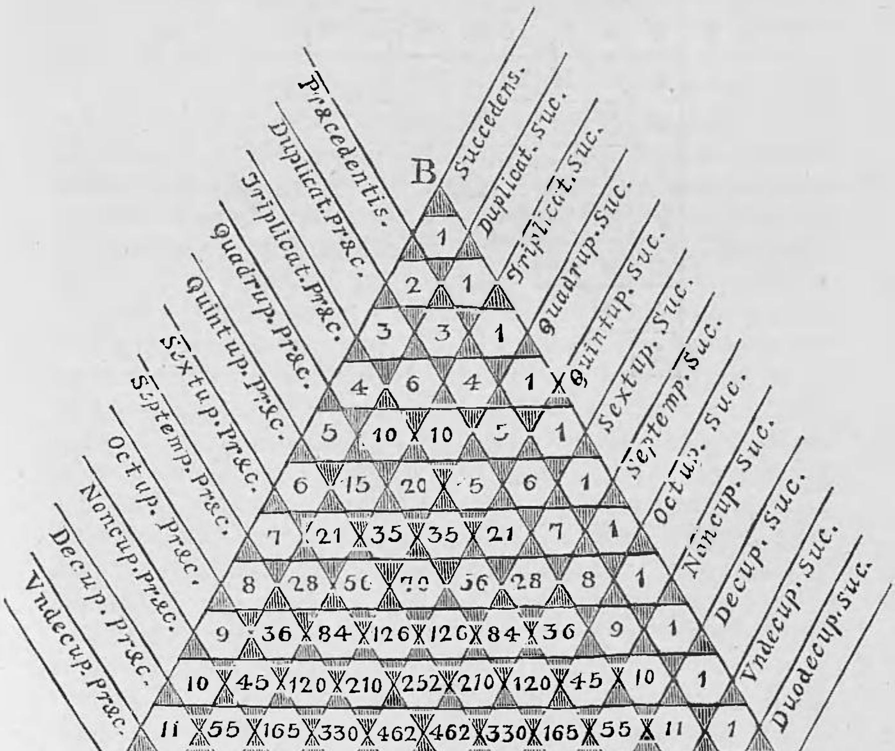
Of the extraction of roots it has been observed, that among all the questions which the development of our ideas of number places in review before us, there is none which, independently of the importance of the solution, has a greater tendency to excite the curiosity of every mind born for calculation. It is comparatively easy to raise roots to powers, but when we demand the roots back again it is not so easy to obtain them. Accordingly, the 7th chapter of the second book of Napier's manuscript digest of Logistic, is entitled, "Of finding the rules for radical extraction." And here the Scotch mathematician, a century before the time of Newton, is disclosed to us on the very track of the famous Binomial Theorem. "Every root," he says, "has its own appropriate and particular rule of extraction. Each rule of extraction consists in resolving the radicate (*radicatum*, Napier's term for *power*) into its supplements (*in sua supplementa*). The supplement is the difference between two *radicates* of the same species. Thus, 100 and 144 are both *duplicates* (Napier's term for the *square*), the one of 10, and the other of 12 ; and the difference between them is 44, which is the true *supplement* of the foresaid *radicates*. Supplements are as various, therefore, as the varieties of the species of radicates (powers) and roots. There is one rule for finding the supplements of *duplication*, and of the extraction of the *bipartient* root ; another of *triplication*, and the extraction of the *tripartient* root ; and so on of all the rest. But MY TRIANGULAR TABLE, filled with *little hexagonal areas*, having, on the right side, a series of units inscribed, and, on the left, a series from unit increasing by unity, and descending from the vertex,—every one of the little hexagonal areas containing within them a number, each equal to the sum of the two numbers placed immediately above it,—teaches the rules for finding the *supplements* of all *radicates* and *roots*."

Napier's directions for drawing the diagram of his Triangular Table.

"Let A, B, C, be a triangle, of which A is the left angle, B the vertical, and C the angle to the right, By so many species of roots as you wish the table to contain, into twice as many parts, and one more, divide each side of the triangle. For instance, in order to extend it to *twelve* species of extractions, let each side of the triangle be divided into *twenty-five* equal parts ; then, beginning from the base, A, C, draw twelve parallel lines within the triangle, connecting the sides by the points in them, alternately taken. In like manner, begin from the side A, B, and draw twelve parallel lines betwixt the alternate points of the base and the side B, C, *extending* the lines beyond the side B, C, about the space of an inch. Exactly in the same manner draw the lines betwixt the side B, A, and the base, extending them an inch beyond B, A. Thus you will have the triangle filled with little hexagonal areas.

"Of these, the twelve to the right, and next the line B, C, must each have a unit inscribed within it. Those on the left must have the numbers 1, 2, 3, 4, &c., as far as 13 (exclusive), successively inscribed within each, descending in their order from the vertex B, to the angle A. Then each interior hexagonal, still vacant, must have inscribed within it *the sum* of the two numbers which are immediately above it. Thus, under 2 and 1 must be written 3 ; under 3 and 3, 6 ; under 3 and 1, 4 ; and so on down to the heel of the table. Lastly, the table must be titled. On the *left* side, above the second hexagonal (2) let there be written, *præcedentis ;* above the third hexagonal (3) write, *duplicatum præcedentis* ; and so on, as far as *duodecuplicatum.* On the right hand of the table, write above the first hexagonal *succedens* ; above the second, *duplicatum succedentis* ; above the third, *triplicatum succedentis ;* and so on, down to *tredecuplicatum.* Just as you have here in the diagram of the table itself, written below.

[*Fac-simile of the diagram in Napier's manuscript, circa* 1590.]



"To every supplement, two parts of the root correspond ; the one part consisting of one or more left-hand figures, already found, and which is called *præcedens* ; the other consisting of a single figure immediately on the right, which is to be sought for, and this is called *succedens.* The supplement, and these parts of the root, mutually compose each other, and are built up together, as will afterwards appear." (From the original Latin MS. in the possession of Lord Napier.)

The mathematical student may compare the above diagram by Napier, about the year 1590, with the following diagram of Pascal's famous Arithmetical Triangle, of which he wrote in the year 1653, more than half a century after Napier's, and of which it is, that Bernoulli writes :—"Nous avons trouve *ce merveilleux theoreme* aussi-bien que Mr. Newton, d'une maniere plus simple que la sienne : Feu *M. Pascal* à ete *le premier qui l'a inventee.*"

Diagram of Pascal's Triangle, circa 1653.

1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	
1	3	6	10	15	21	28	36		
1	4	10	20	35	56	84			
1	5	15	35	70	126				
1	6	21	56	126					
1	7	28	84						
1	8	36							
1	9								
1									