# Tree-Ring Analysis of Timbers from 17 and 19 St Mary's Chare, Hexham, Northumberland 

A J Arnold, R E Howard and Dr C D Litton
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# Tree-Ring Analysis of Timbers from 17 and 19 St Mary's Chare, Hexham, Northumberland 

A J Arnold, R E Howard and Dr C D Litton

## Summary

Forty-three samples were obtained from timbers of the street front and rear range roofs of both numbers 17 and 19 St Mary's Chare, Hexham. Of these 43 samples, 39 were analysed by tree-ring dating, this analysis producing two site chronologies. The first site chronology comprises 33 samples having a combined overall length of 154 rings, these dated as spanning the years AD 1536 to AD 1689.

The second site chronology comprises two samples with an overall length of 79 rings. This second site chronology cannot be dated.

Interpretation of the sapwood on the dated samples would indicate that the roofs of both the front and rear range of number 17 are constructed of timbers felled in $A D$ 1682. It is further indicated that the roofs of the front and rear range of number 19 are both constructed of timbers felled a few years later in AD 1689.

## Keywords

Dendrochronology
Standing Building


#### Abstract

Author's address Department of Mathematics, University of Nottingham, University Park, Nottingham, NG7 2RD.


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## Introduction

Numbers 17 and 19 are a pair of inter-related town houses on the west side of St Mary's Chare, formerly one of the principal streets of the town of Hexham, (NY 396 640; Fig 1). Both houses are similar to each other with number 17, the northern of the two, comprising a four-bay, two storey, north-to-south block, with attics, fronting the street. Number 19 is similar, but this roof comprises four cruck trusses forming a five-bay roof. Illustrations of the facades of each are shown in Figures 2 and 3. Both houses have four-bay rear ranges extending back, east-to-west, into their burgage plots. These rear yards adjoin the precinct of the medieval Augustinian Priory the wall of which forms the western boundary of the site.

The two houses are both generally believed, on stylistic evidence, to be of late seventeenth-century date, although an AD 1992 report by the Royal Commission on the Historical Monuments of England, suggests that number 19 is of the early-eighteenth century. The form of the roof of each part of the building, the two front ranges and the two rear ranges, although having large degrees of similarity, do show some differences.

The crucks of the front range roof of number 17, for example, are jointed. This means that they are made from two pieces of timber, a post and a principal rafter, rather than from a single, curved, piece. The trusses of this roof have collars and carry double purlins. The crucks in the roofs of the other three ranges appear to be made from single curved timbers in the form of true crucks. Furthermore, whilst the front range of number 17, and both the front and rear range roofs of number 19 employ double purlins, the rear range roof of number 17 has only single purlins.

A further difference is to be seen in the carpentry, or conversion, of the timbers in these roofs. The timbers of the front range roof of number 17 are generally very squarely and neatly cut. The joints are also tight fitting, the assembly of them being marked in very clear modern looking Arabic numerals. The timbers in the other three roofs are less neatly cut, much more uneven, and are altogether much more crude looking. The jointing of these roofs tends to be noted in larger Roman numerals.

## Sampling

Sampling and analysis by tree-ring dating of the timbers of 17 and 19 St Mary's Chare were commissioned by English Heritage, this being requested to inform an ongoing programme of repair and conservation work. The English Heritage brief called for the sampling of roof timbers from four specific areas, there being no timbers other than those in the roofs available elsewhere in the building.

Firstly the brief requested the sampling of roof timbers of the four-bay frontage block of number 17, comprising the three sets of well carpentered jointed upper crucks, with collars and double purlins. Secondly timbers of the rear range roof of number 17 were also to be sampled. This roof also consisted of three pairs of upper crucks, although these were stylistically different to those of the front range. Samples were also to be obtained from the roofs of the front and rear ranges of number 19, both roofs consisting of upper crucks, the front range of four trusses, the rear range of three trusses.

For the purposes of tree-ring analysis, given the slight stylistic variations between the roofs, and thus the possibility of different construction dates, each roof area was treated as an individual site with sufficient samples obtained from each for reliable analysis. Thus from these sets of timbers a total of 43 core samples was obtained, with the samples being distributed fairly evenly through the four areas under consideration. Each of these samples was given the code HEX-A (for Hexham, site "A"), and numbered 0143. Thirteen samples, HEX-A01-A13 were obtained from the front range of number 17, with a further ten samples being obtained from the roofs of each of the other three ranges.

The approximate positions of the 43 timbers cored are shown here in Figures $4 a / b$. These figures, provided by English Heritage, are based on drawings made by Kevin Doonan Architects, and amended by Peter Ryder. The exact positions of the timbers are not shown. Details of the samples are given in Table 1. In this report the timbers have been numbered and described from north to south, or east to west, as appropriate.

Timbers were selected for sampling on the basis of their appearing to be original or related to their respected phases, and in appearing to have sufficient rings for satisfactory analysis by tree-ring dating. Timbers were also selected on the basis of their having sapwood or at least the heartwood/sapwood boundary.

The Laboratory would like to take this opportunity to thank Alan Graham, site agent, for arranging access to the site and for helping during sampling. We would also like to thank "Bodyworks" Health and Beauty Parlour, of number 19 for being so helpful and accommodating in assisting with sampling, despite the inconvenience caused. We must also thank the owner of The Clock Shop, who allowed us unhindered access to private apartments to the rear of number 17.

The Laboratory must once again thank both Peter Ryder, Historic Buildings Consultant, and Martin Roberts of English Heritage north-east office for allowing us to use their drawings and descriptions in the introduction to this report.

## Analysis

Each of the 43 samples obtained was prepared by sanding and polishing. It was seen at this point that four samples, HEX-A12, -A13, -A14, and -A34, had too few rings for satisfactory analysis and these were rejected. The annual growth-ring widths of the remaining 39 samples were measured, the data of these measurements being given at the end of the report. These data were then compared with each other by the Litton/Zainodin grouping procedure (see appendix).

At a minimum $t$-value of 4.5 two site chronologies could be created. The first, HEXASQ01, comprises 33 samples, the relative position of the cross-matching samples being shown in the bar diagram, Figure 5. This site chronology has a combined overall length of 154 rings. The second site chronology, HEXASQ02, comprises 2 samples, the relative position of the cross-matching samples being shown in the bar diagram, Figure 6. This site chronology has a combined overall length of 79 rings.

Both site chronologies were compared with a large number of reference chronologies
for oak. This indicated a cross-match for site chronology HEXASQ01 only with a number of these when the date of its first ring is AD 1536 and the date of its last ring is AD 1689. Evidence for this dating is given in the $t$-values of Table 2.

Both site chronologies were also compared with each other, and with the four remaining measured but ungrouped samples, but there was no further satisfactory cross-matching. These four ungrouped samples were then compared individually with a full range of reference chronologies, but again there was no cross-matching, and these samples must remain undated.

## Interpretation

Analysis by dendrochronology has produced a single dated site chronology comprising samples from 33 timbers, with a combined overall length of 154 rings. This site chronology is dated as spanning the years AD 1536 to AD 1689.

Five samples from the front range roof of number 17, HEX-A01, -A02, -A04, -A08, and A11, and one sample from the rear range of number 17, HEX-A16, retain complete sapwood. In each of the six cases the last measured ring date is the same, AD 1682. This is thus the felling date of the timbers represented. The relative positions of the heartwood/sapwood boundaries on the other dated samples from these two roofs would suggest that it is highly probable that these other timbers were felled at this time too. There is certainly no structural indication that the two roofs are other than of a single build.

Three samples from the front range roof of number 19, HEX-A26, -A27, and -A30, and one sample from the rear range of number 19, HEX-A35, also retain complete sapwood. In these cases the last measured complete sapwood ring date is slightly later at AD 1689. The relative positions of the heartwood/sapwood boundaries on the other dated samples from these two roofs would again suggest that it is highly probable that these other timbers were felled in AD 1689 too. Again there is no structural evidence that the two roofs are other than of a single build.

## Conclusion

Analysis by dendrochronology has produced a single dated site chronology comprising samples from 33 timbers, with a combined overall length of 154 rings. This site chronology is dated as spanning the years AD 1536 to $A D 1689$.

Samples with complete sapwood have been obtained from each of the four roof ranges under consideration. This means that the samples have the last rings produced by the trees they represent before they were felled.

This tree-ring analysis indicates that the roofs of both the front and rear ranges of number 17 are constructed of timbers felled in AD 1682, whilst the roofs of the front and rear ranges of number 19 are both constructed of timbers felled a few years later in $A D$ 1689.

Tree-ring dating has thus confirmed the general late-seventeenth century date attributed to these buildings, by giving a much more precise date for the felling of the timber, and showing that the two buildings were in fact constructed a few years apart. This programme of dendrochronology has refuted an earlier suggestion that number 19 might be of early-eighteenth century date and in doing so shown the value of tree-ring dating, even where a general date has been attributed on stylistic grounds.

Six measured samples remain undated, HEX-A20, -A21, -A32, -A40, -A42, and -A43, though two of these, -A42 and -A43 do cross-match with each other. Some of these samples have low numbers of rings, and a few samples, HEX-A32, and -A42 / -A43 for example, have growth rings which show narrow bands, possibly brought about by stressful growing conditions. It is likely that these factors account for these samples not cross-matching with the others and dating.

Three observations might be made about the material from this site. The first is that the $t$-values of the cross-matching of the individual samples tend to suggest that, whilst all the timber used may have come from the same general woodland source, the timber used within each distinct roof has come from a more localised stand or copse. Values in excess of $t=6$ and $t=7$ are found between samples from different roofs, but some values in excess of $t=10$ and $t=11$, are seen between samples within roofs. Some samples, HEX-A42 and -A43 for example, are from timbers probably derived from the same tree.

The second observation concerns the total number of sapwood rings found on some of the samples. The usual $95 \%$ confidence limit for the number of sapwood rings on mature oaks from this part of England is in the range 15 to 40 rings. It will be seen from Table 1 and the bar diagram Figure 5, that a number of samples have less than 15 sapwood rings, although the sapwood on them is complete. The lowest figure found is 11 sapwood rings, on sample HEX-A08, with others having $12-14$ sapwood rings to complete. The greatest number of sapwood rings may be found on sample HEX-A15 which, based upon its being felled in AD 1682, would have had a maximum of 29 sapwood rings.

The final observation concerns the reference chronologies used in Table 2 for dating. It will be seen from this Table that some of these are from areas other than the north of England, from Derbyshire and Nottinghamshire for example. This use of more widespread reference chronologies is due to the fact that there are few, if any, reference chronologies available for northern England that cover the late- sixteenth and seventeenth centuries. In this respect the material from 17 and 19 St Mary's Chare is particularly valuable in providing data for a poorly represented period.

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Table 1: Details of samples from 17 and 19, St Mary's Chare, Hexham, Northumberland.

| Sample number | Sample location <br> Front range number 17 | Total rings | *Sapwood rings | First measured ring date | Last heartwood ring date | Last measured ring date |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HEX-A01 | East lower purlin, north gable - truss 1 | 117 | 13 C | AD 1566 | AD 1669 | AD 1682 |
| HEX-A02 | West lower purlin, north gable - truss 1 | 114 | 14 C | AD 1569 | AD 1668 | AD 1682 |
| HEX-A03 | East wall post, truss 1 | 72 | 14 | AD 1608 | AD 1665 | AD 1679 |
| HEX-A04 | West lower purlin, truss 1-2 | 130 | 15C | AD 1553 | AD 1667 | AD 1682 |
| HEX-A05 | West principal rafter (cruck), truss 1 | 73 | no h/s | AD 1566 | ------ | AD 1638 |
| HEX-A06 | East principal rafter (cruck), truss 2 | 108 | 13 | AD 1562 | AD 1656 | AD 1669 |
| HEX-A07 | West principal rafter (cruck), truss 2 | 71 | no h/s | AD 1560 | ------- | AD 1630 |
| HEX-A08 | West wall post, truss 2 | 96 | 11C | AD 1587 | AD 1671 | AD 1682 |
| HEX-A09 | East lower purlin truss 1-2 | 136 | 6 | AD 1536 | AD 1665 | AD 1671 |
| HEX-A10 | East principal rafter (cruck) truss 3 | 103 | h/s | AD 1569 | AD 1671 | AD 1671 |
| HEX-A11 | East post, truss 3 | 111 | 12C | AD 1572 | AD 1670 | AD 1682 |
| HEX-A12 | West principal rafter (cruck), truss 3 | nm | --- | ------- | ------- | ------ |
| HEX-A13 | East lower purlin, truss 1 - south gable | nm | --- | ------- | ------ | ---- |
|  | Rear range number 17 |  |  |  |  |  |
| HEX-A14 | North cruck blade, truss 1 (east end) | nm | h/s | ------ | ------- | ------ |
| HEX-A15 | South cruck blade, truss 1 | 104 | 27 | AD 1577 | AD 1653 | AD 1680 |
| HEX-A16 | North cruck blade, truss 2 | 110 | 13C | AD 1573 | AD 1669 | AD 1682 |
| HEX-A17 | South cruck blade, truss 2 | 122 | 5 | AD 1552 | AD 1668 | AD 1673 |
| HEX-A18 | North stub tie, truss 2 | 109 | 17 | AD 1564 | AD 1655 | AD 1672 |
| HEX-A19 | South stub tie, truss 2 | 97 | h/s | AD 1559 | AD 1655 | AD 1655 |
| HEX-A20 | North cruck blade, truss 3 | 62 | $\mathrm{h} / \mathrm{s}$ | ------- | ------ | ------ |
| HEX-A21 | South cruck blade, truss 3 | 54 | h/s | ----- | ------ | ----- |
| HEX-A22 | North stub tie, truss 3 | 65 | no h/s | AD 1572 | ------ | AD 1636 |
| HEX-A23 | South stub tie, truss 3 | 97 | 15 | AD 1574 | AD 1655 | AD 1670 |

Table 1: Continued

| Sample number | Sample location <br> Front range number 19 | Total rings | *Sapwood rings | First measured ring date | Last heartwood ring date | Last measured ring date |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HEX-A24 | East cruck blade, truss 1 | 114 | 15 | AD 1570 | AD 1668 | AD 1683 |
| HEX-A25 | East lower purlin, truss 1-2 | 86 | 13 | AD 1597 | AD 1669 | AD 1682 |
| HEX-A26 | East cruck blade, truss 2 | 82 | 20C | AD 1608 | AD 1669 | AD 1689 |
| HEX-A27 | Collar, truss 2 | 99 | 25C | AD 1591 | AD 1664 | AD 1689 |
| HEX-A28 | East cruck blade, truss 3 | 81 | 14 | AD 1606 | AD 1672 | AD 1686 |
| HEX-A29 | West cruck blade, truss 3 | 106 | 7 | AD 1569 | AD 1667 | AD 1674 |
| HEX-A30 | Collar, truss 3 | 66 | 20 C | AD 1624 | AD 1669 | AD 1689 |
| HEX-A31 | East cruck blade, truss 4 | 89 | 13 | AD 1592 | AD 1667 | AD 1680 |
| HEX-A32 | Collar, truss 4 | 55 | no h/s | ------ | ------ | ----- |
| HEX-A33 | West principal rafter, truss 4 | 131 | 16 | AD 1549 | AD 1663 | AD 1679 |

Rear range number 19

| HEX-A34 | South cruck blade, truss 1 | nm | --- |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HEX-A35 | South purlin, truss 1-2 | 101 | 20 C | AD 1589 | AD 1669 | AD 1689 |
| HEX-A36 | North cruck blade, truss 2 | 88 | 8 | AD 1595 | AD 1674 | AD 1682 |
| HEX-A37 | South cruck blade, truss 2 | 112 | 7 | AD 1570 | AD 1674 | AD 1681 |
| HEX-A38 | Collar, truss 2 | 131 | 18 | AD 1553 | AD 1665 | AD 1683 |
| HEX-A39 | North cruck blade, truss 3 | 85 | 17 | AD 1599 | AD 1666 | AD 1683 |
| HEX-A40 | South cruck blade, truss 3 | 54 | no h/s | ------- | ------ | ------- |
| HEX-A41 | Collar, truss 3 | 90 | no h/s | AD 1561 | ------- | AD 1650 |
| HEX-A42 | North purlin, truss 3 to west gable | 77 | no h/s | ------ | ------- | ------ |
| HEX-A43 | South purlin, truss 3 to west gable | 78 | no h/s | ------ | ------ | ------ |

*h/s = the heartwood/sapwood boundary is the last ring on the sample $\mathrm{nm}=$ sample not measured
$\mathrm{C}=$ complete sapwood retained on the sample, the last measured ring date is the felling date of the timber

Table 2: Results of the cross-matching of chronology HEXASQ01 and relevant reference chronologies when the date of the first ring is AD 1536 and the last ring date is $A D 1689$

| Reference chronology | Span of chronology | $t$-value |  |
| :--- | :---: | :---: | :--- |
|  |  |  |  |
| England | AD $401-1981$ | 7.5 | (Baillie and Pilcher 1982 unpubl) |
| Scotland | AD $946-1975$ | 7.2 | (Baillie 1977) |
| Rufford Mill, Notts | AD 1554-1744 | 7.0 | (Laxton and Litton 1988) |
| Staircase House, Stockport, Greater Manchester | AD 1489-1656 | 6.7 | (Howard et al 2003) |
| 15/17 St John's Street, Wirksworth, Derbys | AD 1586-1676 | 5.7 | (Howard et al 1995) |
| Brewhouse Yard Museum, Nottm | AD 1544-1701 | 5.2 | (Howard et al 1994 ) |
| East Midlands | AD $882-1981$ | 4.5 | (Laxton and Litton 1988) |

Figure 1: Map to show general location of St Mary's Chare

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Figure 2: Front elevation of 17 St Mary's Chare


Figure 3: Front elevation of 19 St Mary's Chare


Figure 4a: Plan to show approximate position of sampled timbers from number 17


Figure 4b: Plan to show approximate position of sampled timbers from number 19


Figure 5: Bar diagram of the samples in site chronology HEXASQO1, sorted by sampling location in last measured ring position

white bars $=$ heartwood rinas, shaded area $=$ sapwood rinas
$\mathrm{h} / \mathrm{s}=$ heartwood/sapwood boundary is last ring on sample
$\mathrm{C}=$ complete sapwood retained on sample, the last measured ring date is the felling date of the timber

Figure 6: Bar diagram of the samples in site chronology HEXASQ02

white bars = heartwood rinas
$\mathrm{h} / \mathrm{s}=$ heartwood/sapwood boundary is last ring on sample

Data of measured samples - measurements in 0.01 mm units

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HEX-A01A 117
    154171179230230196169172221184157151117141132 99132138171161
    173194155160197168192236263228169142112 95128112140151154148
    129137136136136153149199176183111168112140120122107 9977 74
    122121102115107131107103115140163175 186 142141187111 54110 127
    117 89 102 53 62 59 71 67116 91 108 62 54 56 63 63 84 69 76 86
    829085 95 101 87 133 122 8972 95 116140136 91 121 139
HEX-A01B }11
    137166167233239227177174219184153152110133134101127139172165
    170178132173187162204240286242169139117 92128131133146165134
    14613613413713016214917417419110916511114612711810210071 78
    12412798110113124104112119133165178184135154183107 61109121
    130 93 98 48 6260 69 74 122 81 112 56 55 56 63 59 87 73 78 75
    9079 99 77 115 90 125119 87 73 102122141124 95 121 132
HEX-A02A 114
    149186262181206316267181235258369358314160142175278220199 116
    152158131180198137165115 6843 35564672100 90 86 75 54100
    94 92 71 103111109 73 48 63100 84 92 90 83 63 40 42 57 77 83
    89 99 71 63 33 26 44 36 76 12369 99 74 34 31 43 41 73 88 87
    977258 55120131137174103 81 77 99121115 138 96 101 83 72 72
    1131411051101461246789 92 87125696782
HEX-A02B 114
    106172272195207282196140225228373363309156135178283256206110
    164158144185202132155110 80 39 36 56 71 57116 97 94 75 59 85
    82 93 84 88 104113 74 50 65 93 72 97 91 80 63 39 46 55 80 86
    96 98 73 65 32 36 26 50 71 120 66 105 70 32 29 40 39 79 81 93
    10063 71 491211361381629884 82 91 120114149 84107 83 70 64
    991201031281401247680 98 93130646871
HEX-A03A }7
    979511610914519422016096123110 106 105110113 91 72 96 74 82
    711219092128 79 6286 90114156 8711811450 3966 80126106
    111111110116114168250282257183132135145133126108124161204131
    109147189166175197153 79102121103107
HEX-A03B }7
    116 94125 9614318822313795117113113101118 98 77 85 89 75 83
    7312884921218163 81 9811115688121105 51 45 63 82 119109
    112130127102128164273270258174147139132136121109121160219134
    9813221814917119614080 93 127 99137
HEX-A04A 130
    184170134207210200161210152136115 94 50 85 79 91 73 102 80 85
    697877 34 394563 98 70 34 6273 91 130117 67132100 75 99
    125150140 98 106 49 28 3644 43 60109 81 102 74 72 46 86 54 65
    88130 98 45 75 5967 80 75 77 76 75 53113106 95 127103 115 71
    644655139165260177 245172794677101177158117 77 444743
    701181111508869 8494100 9713618818116590 74138213223245
29723192130112150231207175227
HEX-A04B 130
172180138204208203162214145133126 90 57 76 88 82 80 94 89 80
76 84 69 34 35 39 65102 81 35 6267 94132123 90115 104 80 95
121149131104115 42 30 30 36 47 55 120 68 102 73 70 55 77 60 65
8412510247 74 67 57 86 74 81 79 61 61 87113 93122109127 66
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61525814416125817624717061588010317815611980414735
8011311414891756910899971291921851658985117222226236 28823395124121142241201171241
HEX-A05A 73
231230190186224274207233184167148139146200200178145162198133
160172124118120111130145176129107891168512311710412298108
$\begin{array}{llllllllllllllllllllll}78 & 76 & 59 & 62 & 86 & 67 & 56 & 76 & 94 & 94 & 56 & 66 & 82 & 70 & 78 & 73 & 122 & 126 & 114 & 95\end{array}$
164186163218166187183151132183247236247
HEX-A05B 73
206231180186266275202209212202157132150183179169165163188141
18317011511513798153153184134119941157912211811010910999
$\begin{array}{llllllllllllllllllll}86 & 78 & 69 & 73 & 82 & 64 & 61 & 73 & 101 & 83 & 66 & 80 & 57 & 83 & 90 & 71 & 154 & 121 & 108 & 102\end{array}$
155193136181180177189142118168243239242
HEX-A06A 108
281435369182267369379247203162146220234352184159254267174128


$\begin{array}{lllllllllllllllllllllll}35 & 67 & 52 & 47 & 31 & 27 & 28 & 28 & 40 & 27 & 30 & 31 & 27 & 42 & 65 & 127 & 200 & 257 & 284 & 190\end{array}$

144286261372371221274365
HEX-A06B 108
282431367183259368389236205169143209243355174172248273175123
1171281721492021851061361509811813314493678069588457
$\begin{array}{lllllllllllllllll}67 & 54 & 64 & 56 & 42 & 76 & 72 & 49 & 43 & 66 & 92 & 97 & 62 & 60 & 37 & 34 & 43 \\ 34 & 45 & 50\end{array}$

55466561971111821198869525886156254203243235200171
160287314337366209246332
HEX-A07A 71
15886100103792297202118104133207151230293218139144176200
246144609613017822923518221911293157211270227182193172139
21416916026723127624622321724227520624524724624693180174188
210224198172108172204247288269259
HEX-A07B 71
1781111091079223100199124110119212137239282231133148176195 2491455910212918622723518421711790161224275222188175173133 21817816026523426624121620822328920524625222925390181187194 216220197192113169217241270277209
HEX-A08A 96
162161159180179167238236205197198190136166161166156198147145 107154117132111166193191205128168175185180176216182171160177 212220219153179198169142155196216252172200212156103118152200 158114125109119127186210218254167154174172136203156147191157 156223234307194260231194157190178227204165166189 HEX-A08B 96
125152154181169178239216245185182196145172142180157180165151 111143117144118169224174184123175177204172185201206174154163 21221522415717520116313714920821024717319121516990116156191 151118120112114132186217217246171154172167145207158139198156 152227225315193260234200152190176198230163166211
HEX-A09A 136
1229175117238202283309225264228204201288313301222231183193 1991922622401441931581551721031081181221341071158913312596
$\begin{array}{llllllllllllllllll}79 & 86 & 86 & 85 & 93 & 94 & 66 & 62 & 86 & 72 & 81 & 69 & 54 & 58 & 74 & 48 & 65 & 72\end{array} 6985$
$\begin{array}{lllllllllllllllllll}45 & 40 & 40 & 58 & 50 & 59 & 50 & 69 & 62 & 85 & 58 & 73 & 67 & 50 & 51 & 52 & 53 & 42 & 67\end{array} 53$

$\begin{array}{lllllllllllllllllll}31 & 55 & 62 & 42 & 53 & 50 & 41 & 40 & 53 & 56 & 74 & 57 & 60 & 49 & 58 & 46 & 47 & 70 & 71 \\ 64\end{array}$ 9149647064718512012615313464828487125
HEX-A09B 136
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193217253240128185151151163881191131251241181179912712899
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HEX-A10B 103
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HEX-A11B 111
203218213194182127174186235169157189194173216165118140131102 138154191158142127124861291151061371221131077310292146115 14219919616296180207210208184237201164165208226225214149170 16811995149240216275197175225177801051302021231018710067 7510313214516011011610410281126118971039181144162283194 171156136869111315212067105116
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678610791
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    159174135 75 119107102118132120 97 63 62 49 66 75 116 84 61 72
    35 39 40 50 80 111100 108 136 59 45 64 84 159 133127 120 113 76 80
    123248157183161212200230232278243165267326203181244303256260
    287254193211150167207206177187
HEX-A16B 110
32734323592111167145203171142169205194319272205226231147264
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165177135 73 121 103 103 118 133125 95 68 64 45 70 69 123 95 49 77
47 37 40 56 73120 96 119129 71 35 61 114 136 150120 120141 79 65
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    126 157164153 81 58 34 30 44 51 42 81 129 170 122 123 126 96 108 72
    124158 208 184 129172174 133183167133128100 64 52 50 52 80 78 44
    52 35 25 36 31 46 94 98 126161 59 42 44 81 132136109106106 66
    59115196205240 94114 87115116144197127177 205142145237269236
252288
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    120154169155 82 64 31 26 3963 41 74128171125115134101113 83
    114157 205 200 108 175 186 128 178162145135 80 72 50 55 43 87 67 56
    47 36 21 30 31 59 84103127 160 61 40 49 81 128 133110106 99 77
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144 75 96 102 79 59 65 59 74 73 79 73 62 89 103 69 67 71 72 72
56}6744350 53 73 61 44 41 23 24 333 33 57 71 30 56 54 42 27,
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30 61 41 39 50 81 85 98 132
HEX-A19A 97
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65112 94 72 47 52 84 100 78 83 73 91 97 57 76 109 160
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HEX-A19B 97
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HEX-A20A 62
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291389412372333382511590638449371531407301373321375463443283 344322
HEX-A20B 62
129122205229181190188254184222199346345262243228172146172207 22836027425219619914990112183420307500459203306263377394305 302387414350331404511587637449364523414305373326395465437294 332334
HEX-A21A 54
17415313893128977644721041952302251931421309752111121 335342368370118178224276286246211302252268216186313358441263 227441385259351345466553450243265287280267
HEX-A21B 54
 274317363366103201219266287232230297255263212172308344430288
244437366267367310449532480227272272256250
HEX-A22A 65
297419494276143213393439443327230192233196222255193205193151 18218928521619511610279148140124166190192171161183157166142 $18014716315712210113110412813099100636455 \quad 55 \quad 58 \quad 789578$ 6543517458
HEX-A22B 65
286408493270151217392432509326236179239206219232190206197160 18919228220519212687100130147123161211192176164183174166145 $1721561601631131061251141261269796755356 \quad 6151709771$ 6645775445
HEX-A 23 A 97
449292137196362376445364259249250202212243176195183138159234
222185172851069293122117130168171181160172150158143162160



HEX-A23B 97
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127180177 197228226 160204176143151153 123 58 46 68 82 96 98 134
120122105674567122160250184223253115100162230251151167176
191193 145 213 93 93 85 67 54 43 49 50 70 84 58 81 78 76 125 104
164118137114 98 83 86 153 142163121131130136
HEX-A25A 86
323193180228209159189268166180217166146208173 236182193 250 211
247223214227162184 92 118115 164178131227158144114 73 86 98 123
21027313617618611995119154237195150124107 90 98132162222276
216 22121217514719820619623119510717213319816817315114410894
76153192125166196
HEX-A25B }8
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222217202178153188196213224176114166155199171173168159101 56
    84144216118153193
HEX-A26A 82
    159184156177170196234222116157192183194190171 88102144163141
214216128121119991 68125179160174119162195 64 71 1111178213156
161161154151 116 147 49 43 38849 51 43 34 67 62 94 78 116 104 98
112126158150172175155 80 90163150171113157194195179 98 189220
199183
HEX-A26B }8
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    202220 122 129114 94 78 116177 160171129154 200 66 60115 176 222 132
    168 158 141 156 118 151 44 49 43 44 47 44 42 60 62 94 75 116 96 86
    128122167151 172171154 87105 131172171102175 182185169108178 208
200183
HEX-A27A 99
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    135106100202185197170197178174161150108125157232207215 278169
    137130}723226 33101205117198256103 47 53 159 231134 58 48 37
    33 47 64 52 70 68 68 65 44 68 63 77 79 70 74 77 42 46 66 92
    101 81 108 86 43 55 59 82 93 55 226141141 99120155119195190
HEX-A27B }9
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    127115107198191193157202162177 159153 84133158230210207284163
    131140 78 31 27 30101 221110189261 95 51 58 159 225 128 61 54 32
    37}515154 56 73 63 69 64 45 76 58 77 75 72 76 76 46 38 75 90
    102 83 111 79 43 60 62 84 92 65 88 118138106143191 99188181
HEX-A28A 81
    145169178142177 151183188164183162173 202150168135 225 97 79 111
    113145175 171137123141 81 60104 131155 225127173 172 1401111110131
    171179152143134164132174179206225159203149198193186185147184
    158143107157203198186203171127 72 144184190140122177160173135
    125
HEX-A28B }8
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    111144178182136128129 94 56125107168 207135161191123111102138
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    166140106161193201198196169118 72147189186138124176170155149
121
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    114 52 121 105 125 135173 69 149129124104104 87 36 80 60103 72 120
    156 70 74 56 47 29 39 62 90 129 88 104 129 48 39 84 107 113 79 79
    84 66 85 68 82 103 94 136 59 72 61 96 89 140 152 121 107 105 64 87
    8112277121 87111
HEX-A29B }10
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130125 82 127132150110 62 89 55 41 89 108 77 116 127120105 124 97
108 61 120 112 121 125 175 79 148 134 123 100 99 86 36 67 61 99 78 124
16070 84 51 50 29 36 62 92 127 87 101 137 48 37 87 109 112 77 86
84 72 89 58 89 97 107 129 57 74 64 84 91 139152122 115 100 74 77
8612075120 82136
HEX-A30A 66
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    61 167180 130 93 70 44 36 47 47 27 50 53 51 49 42 59 62 77 122
    841211256358106233348277289163 741111120213212136227233178
    116184163173135122
HEX-A30B }6
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    66173 182 132 92 63 53 33 37 40 32 57 54 44 55 37 48 49 80 111
    88124119 71 57105233 35327727016579114112216208139230222181
    114184168173135116
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192224248323159251257262252256251146 148173207182182212173152
150120 9713217320122015119922790 96 128172210142146159152145
13918254 38 42 51 56 49 79 97 168 177 139 178 124 94 152 119 129 123
164 8412860126136163140118
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27 28 14 10018 37 22 28 13 13 24 16 20 25 41
HEX-A32B 55
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214208203215282162189208234231176 187 63 55 100 115 32 19 25 20
23
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178135121168183185146100125 77 66110131112142152113108136 98
116 77 109131141140190 94 171 161 145137142104 63 55 77 121 117 130
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109 92 116 105 145 90 83 94 50 80 76 75 84 135 154113 116 116 69109
971249514970103 58 83100 99104
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    120 80 113 133137145 170 82 177 153145147154 99 58 65 71 120119 128
    140 91 98 70 40 40 34 62 135190109134154 66 39 98 122 160119103
    105 97 114105145 93 71 107 40 95 70 74 85 143 159 112 124105 80 113
    96124 83 132 78 103 58 80104104104
HEX-A35A 101
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    107142135125 155 167122 121 125 152 166 159 184 137 79 67 58 62 78 99
    170185127104 49 30 90 145171232150207 259 86 65 43 78 108 119 122
    100 62 56 84 91 144138190141 95 75 88 82 108 125118 128 98 89 56
    93200172218177176 86 95104115127 94120160159138 69 91 169130
    1 1 9
HEX-A35B 101
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    104 144132129151167120123129150157164177133 79 65 55 70 76 102
    168182124110 52 29 90142167224150208 256 96 61 41 76 122 111116
    10364 63 79 8714113619115280 73 89 93 104 126 121 128 98 87 68
    89215159215183173 90 86 89 115123 92125151172117 89 89 167 128
    124
HEX-A36A 88
321247332204187263247294371410328303 300239253230194238196218
229142171184161163162186125108154160159177 255154182107107104
136 177 181 221 50 42 29 31 37 32 51 63 60 54 50 49 588 57 69 54
6265 81 103 93103118154126151127110 78 119113165107131126115
8183118105846788124
HEX-A36B 88
339236 329228184242253279375415309301321226254246193226 207189
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134194 177 231 44 44 33 27 28 40 53 63 60 58 46 54 52 588}7
55758493 9610810015413414811712287114118168100126122134
68 8610114075 53 83131
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89 93 60 53 60 77 103 114 186 45 63 61 46 48 44 62 94 87 93 81
80103128134120100114 96 101 89110 87 122117123129 85 76 94 110
162116156141118134101110123 81 92 90
HEX-A37B 112
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156110163140129114125113151 90 77 91
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186186 215 178 156 214166 170130117115 90 55 51 82 64 83 93 95 92
133101 83 96 70 89 107 120 97 94 68 108 84 101 106 76 96 84 85 88
100109 113 92 85 68 55 81 95 79 93 85 95 88 110106 92 66 76 84
59 81 109 53 84 82 79 81 83 65 58 45 70 70 65 70 88 68 68 52
42}38851[66 71 104 63 98 76 35 37 33 51 67 57 54 76 69 62 50
67}65949997870 74 64 68 82 102 76 96 73 51 93 79 102 64 87
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HEX-A39A 85
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226212159177217317249273193226277318340424414351224243396416
407278239377369
HEX-A40A 54
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340338245234341397289330398283113173238401
HEX-A40B 54
1081692181837717719121610013272104174234267297273276318275
2593213352872031198485201270235227218309247379294286287367
373328227223337446279328393301124188223415
HEX-A41A 90
2072873002108574809311416694140162206107777565127132

$\begin{array}{lllllllllllllllll}95 & 61 & 95 & 142 & 88 & 84 & 86 & 65 & 58 & 74 & 67 & 93 & 82 & 59 & 66 & 64 & 70 \\ 78 & 75 & 81\end{array}$
$\begin{array}{llllllllllllllllll}70 & 53 & 38 & 30 & 29 & 44 & 56 & 69 & 90 & 64 & 54 & 44 & 31 & 33 & 45 & 58 & 55 & 59\end{array} 5281$
$\begin{array}{lllllllll}83 & 47 & 29 & 38 & 57 & 37 & 31 & 23 & 32 \\ 39\end{array}$
HEX-A41B 90
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$\begin{array}{lllllllllllllllll}56 & 67 & 37 & 22 & 36 & 47 & 46 & 55 & 95 & 73 & 34 & 52 & 34 & 30 & 54 & 56 & 53 \\ 65 & 47 & 74\end{array}$
$\begin{array}{llllllllll}92 & 53 & 21 & 35 & 58 & 34 & 28 & 31 & 30 & 38\end{array}$
HEX-A42A 77
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17612050324980175211121150173115160145126235211222239128
14795159178122253144891127686113199120275221133
HEX-A42B 77
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47491001449683525353902582631129868413276172274
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477498385520593423601507377484141727062712436302932
 26820613461536512824127916015716511299119144230234203148 79116121119236190246172861289390152241247265220211

# APPENDIX 

Tree-Ring Dating

## The Principles of Tree-Ring Dating

Tree-ring dating, or dendrochronology as it is known, is discussed in some detail in the Laboratory's Monograph, 'An East Midlands Master Tree-Ring Chronology and its uses for dating Vernacular Building' (Laxton and Litton 1988) and, Dendrochronology; Guidelines on Producing and Interpreting Dendrochronological Dates (English Heritage 1988). Here we will give the bare outlines. Each year an oak tree grows an extra ring on the outside of its trunk and all its branches just inside its bark. The width of this annual ring depends largely on the weather during the growing season, about April to October, and possibly also on the weather during the previous year. Good growing seasons give rise to relatively wide rings, poor ones to very narrow rings and average ones to relatively average ring widths. Since the climate is so variable from year to year, almost random-like, the widths of these rings will also appear random-like in sequence, reflecting the seasons. This is illustrated in Figure 1 where, for example, the widest rings appear at irregular intervals. This is the key to dating by tree rings, or rather, by their widths. Records of the average ring widths for oaks, one for each year for the last 1000 years or more, are available for different areas. These are called master chronologies. Because of the random-like nature of these sequences of widths, there is usually only one position at which a sequence of ring widths from a sample of oak timber with at least 70 rings will match a master. This will date the timber and, in particular, the last ring.

If the bark is still on the sample, as in Figure 1, then the date of the last ring will be the date of felling of the oak from which it was cut. There is much evidence that in medieval times oaks cut down for building purposes were used almost immediately, usually within the year or so (Rackham 1976). Hence if bark is present on several main timbers in a building, none of which appear reused or are later insertions, and if they all have the same date for their last ring, then we can be quite confident that this is the date of construction or soon after. If there is no bark on the sample, then we have to make an estimate of the felling date; how this is done is explained below.

## The Practice of Tree-Ring Dating at the University of Nottingham Tree-Ring dating Laboratory

1. Inspecting the Building and Sampling the Timbers. Together with a building historian the timbers in a building are inspected to try to ensure that those sampled are not reused or later insertions. Sampling is almost always done by coring into the timber, which has the great advantage that we can sample in situ timbers and those judged best to give the date of construction, or phase of construction if there is more than one in the building. The timbers to be sampled are also inspected to see how many rings they have. We normally look for timbers with at least 70 rings, and preferably more. With fewer rings than this, 50 for example, sequences of widths become difficult to match to a unique position within a master sequence of ring widths and so are difficult to date (Litton and Zainodin 1991). The cross-section of the rafter shown in Figure 2 has about 120 rings; about 20 of which are sapwood rings - the lighter rings on the outside. Similarly the core has just over 100 rings with a few sapwood rings.

To ensure that we are getting the date of the building as a whole, or the whole of a phase of construction if there is more than one, about 8 to 10 samples per phase are usually taken. Sometimes we take many more, especially if the construction is complicated. One reason for taking so many samples is that, in general, some will fail to give a date. There may be many reasons why a particular sequence of ring widths from a sample of timber fails to give a date even though others from the same building do. For example, a particular tree may have grown in an odd ecological niche, so odd indeed that the widths of its rings were determined by factors other than the local climate! In such circumstances it will be impossible to date a timber from this tree using the master sequence whose widths, we can assume, were predominantly determined by the local climate at the time.


Fig 1. A wedge of oak from a tree felled in 1976. It shows the annual growth rings, one for each year from the innermost ring to the last ring on the outside just inside the bark. The year of each ring can determined by counting back from the outside ring, which grew in 1976.


Fig 2. Cross-section of a rafter showing the presence of sapwood rings in the left hand corner, the arrow is pointing to the heartwood/sapwood boundary $(\mathrm{H} / \mathrm{S})$. Also a core with sapwood; again the arrow is pointing to the $\mathrm{H} / \mathrm{S}$. The core is about the size of a pencil.


Fig. 3 Measuring ring widths under a microscope. The microscope is fixed while the sample is on a moving platform. The total sequence of widths is measure twice to ensure that an error has not been made. This type of apparatus is needed to process a large number of samples on a regular basis.


Fig 4. Three cores from timbers in a building. They come from trees growing at the same time. Notice that, although the sequences of widths look similar, they are not identical. This is typical.

Sampling is done by coring into the timber with a hollow corer attached to an electric drill and usually from its outer rings inwards towards where the centre of the tree, the pith, is judged to be. An illustration of a core is shown in Figure 2; it is about 15 cm long and 1 cm diameter. Great care has to be taken to ensure that as few as possible of the outer rings are lost in coring. This can be difficult as these outer rings are often very soft (see below on sapwood). Each sample is given a code which identifies uniquely which timber it comes from, which building it is from and where the building is located. For example, CRO-A06 is the sixth core taken from the first building (A) sampled by the Laboratory in Cropwell Bishop. Where it came from in that building will be shown in the sampling records and drawings. No structural damage is done to any timbers by coring, nor does it weaken them.

During the initial inspection of the building and its timbers the dendrochronologist may come to the conclusion that, as far as can be judged, none of the timbers have sufficient rings in them for dating purposes and may advise against sampling to save further unwarranted expense.

All sampling by the Laboratory is undertaken according to current Health and Safety Standards. The Laboratory's dendrochronologists are insured.
2. Measuring Ring Widths. Each core is sanded down with a belt sander using medium-grit paper and then finished by hand with flourgrade-grit paper. The rings are then clearly visible and differentiated from each other with a result very much like that shown in Figure 2. The core is then mounted on a movable table below a microscope and the ring-widths measured individually from the innermost ring to the outermost. The widths are automatically recorded in a computer file as they are measured (see Fig 3).
3. Cross-matching and Dating the Samples. Because of the factors besides the local climate which may determine the annual widths of a tree's rings, no two sequences of ring widths from different oaks growing at the same time are exactly alike (Fig 4). Indeed, the sequences may not be exactly alike even when the trees are growing near to each other. Consequently, in the Laboratory we do not attempt to match two sequences of ring widths by eye, or graphically, or by any other subjective method. Instead, it is done objectively (ie statistically) on a computer by a process called cross-matching. The output from the computer tells us the extent of correlation between two sample sequences of widths or, if we are dating, between a sample sequence of widths and the master, at each relative position of one to the other (offsets). The extent of the correlation at an offset is determined by the $t$-value (defined in almost any introductory book on statistics). That offset with the maximum $t$-value among the $t$-values at all the offsets will be the best candidate for dating one sequence relative to the other. If one of these is a master chronology, then this will date the other. Experiments carried out in the past with sequences from oaks of known date suggest that a $t$-value of at least 4.5 , and preferably at least 5.0 , is usually adequate for the dating to be accepted with reasonable confidence (Laxton and Litton 1988; Laxton et al 1988; Howard et al 1984-1995).

This is illustrated in Fig 5 with timbers from one of the roofs of Lincoln Cathedral. Here four sequences of ring widths, LIN-C04, 05, 08, and 45, have been cross-matched with each other. The ring widths themselves have been omitted in the bar-diagram, as is usual, but the offsets at which they best cross-match each other are shown; eg the sequence of ring widths of C08 matches the sequence of ring widths of C45 best when it is at a position starting 20 rings after the first ring of C 45 , and similarly for the others. The actual $t$-values between the four at these offsets of best correlations are in the matrix. Thus at the offset of +20 rings, the $t$-value between C45 and C08 is 5.6 and is the maximum found between these two among all the positions of one sequence relative to the other.

It is standard practice in our Laboratory first to cross-match as many as possible of the ringwidth sequences of the samples in a building and then to form an average from them. This average is called a site sequence of the building being dated and is illustrated in Fig 5. The fifth bar at the bottom is a site sequence for a roof at Lincoln Cathedral and is constructed from the matching sequences of the four timbers. The site sequence width for each year is the average of the widths in each of the sample sequences which has a width for that year. Thus in Fig 5 if the widths shown are 0.8 mm for $\mathrm{C} 45,0.2 \mathrm{~mm}$ for $\mathrm{C} 08,0.7 \mathrm{~mm}$ for C 05 , and 0.3 mm for C 04 , then the corresponding width of the site sequence is the average of these, 0.55 mm . The actual sequence
of widths of this site sequence is stored on the computer. The reason for creating site sequences is that it is usually easier to date an average sequence of ring widths with a master sequence than it is to date the individual component sample sequences separately.

The straightforward method of cross-matching several sample sequences with each other one at a time is called the 'maximal $t$-value' method. The actual method of cross-matching a group of sequences of ring-widths used in the Laboratory involves grouping and averaging the ring-width sequences and is called the 'Litton-Zainodin Grouping Procedure'. It is a modification of the straight forward method and was successfully developed and tested in the Laboratory and has been published (Litton and Zainodin 1991; Laxton et al 1988).
4. Estimating the Felling Date. As mentioned above, if the bark is present on a sample, then the date of its last ring is the date of the felling of its tree. Actually it could be the year after if it had been felled in the first three months before any new growth had started, but this is not too important a consideration in most cases. The actual bark may not be present on a timber in a building, though the dendrochronologist who is sampling can often see from its surface that only the bark is missing. In these cases the date of the last ring is still the date of felling.

Quite often some, though not all, of the original outer rings are missing on a timber. The outer rings on an oak, called sapwood rings, are usually lighter than the inner rings, the heartwood, and so are relatively easy to identify, For example, sapwood can be seen in the comer of the rafter and at the outer end of the core in Figure 2, both indicated by arrows. More importantly for dendrochronology, the sapwood is relatively soft and so liable to insect attack and wear and tear. The builder, therefore, may remove some of the sapwood for precisely these reasons. Nevertheless, if at least some of the sapwood rings are left on a sample, we will know that not too many rings have been lost since felling so that the date of the last ring on the sample is only a few years before the date of the original last ring on the tree, and so to the date of felling.

Various estimates have been made and used for the average number of sapwood rings in mature oak trees (English Heritage 1998). A fairly conservative range is between 15 and 50 and that this holds for $95 \%$ of mature oaks. This means, of course, that in a small number of cases there could be fewer than 15 and more than 50 sapwood rings. For example, the core CRO-A06 has only 9 sapwood rings and some have obviously been lost over time - either they were removed originally by the carpenter and/or they rotted away in the building and/or they were lost in the coring. It is not known exactly how many sapwood rings are missing, but using the above range the Laboratory would estimate between a minimum of $6(=15-9)$ and a maximum of $41(=50-9)$. If the last ring of CRO-A06 has been dated to 1500 , say, then the estimated felling-date range for the tree from which it came originally would be between 1506 and 1541. The Laboratory uses this estimate for sapwood in areas of England where it has no prior information. It also uses it when dealing with samples with very many rings, about 120 to the last heartwood ring. But in other areas of England where the Laboratory has accumulated a number of samples with complete sapwood, that is, no sapwood lost since felling, other estimates in place of the conservative range of 15 to 50 are used. In the East Midlands (Laxton et al 2001) and the east to the south down to Kent (Pearson 1995) where it has sampled extensively in the past, the Laboratory uses the shorter estimate of 15 to 35 sapwood rings in $95 \%$ of mature oaks growing in these parts. Since the sample CRO-A06 comes from a house in Cropwell Bishop in the East Midlands, a better estimate of sapwood rings lost since felling is between a minimum of $6(=15$ 9) and $26(-35-9)$ and the felling would be estimated to have taken place between 1506 and 1526, a shorter period than before. (Oak boards quite often come from the Baltic and in these cases the $95 \%$ confidence limits for sapwood are 9 to 36 (Howard et al 1992, 56)).

Even more precise estimates of the felling date and range can often be obtained using knowledge of a particular case and information gathered at the time of sampling. For example, at the time of sampling the dendrochronologist may have noted that the timber from which the core of Figure 2 was taken still had complete sapwood but that none of the soft sapwood rings were lost in coring. By measuring into the timber the depth of sapwood lost, say 2 cm , a reasonable estimate can be made of the number of sapwood rings lost, say 12 to 15 rings in this case. By adding on 12 to 15 years to the date of the last ring on the sample a good tight estimate for the range of the felling date can be obtained, which is often better than the 15 to 35 years later we would have estimated without this observation. In the example, the felling is now estimated to
$t$-value/offset Matrix


## Bar Diagram

$\begin{array}{llllllllllllll}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 & 90 & 100 & 110\end{array}$

C45


## SITE SEQUENCE



Fig 5. Cross-matching of four sequences from a Lincoln Cathedral roof and the formation of a site sequence from them.

The bar diagram represents these sequences without the rings themselves. The length of the bar is proportional to the number of rings in the sequence. Here the four sequences are set at relative positions (offsets) to each other at which they have maximum correlation as measured by the $t$ values.

The $t$-value/offset matrix contains the maximum $t$-values below the diagonal and the offsets above it. Thus, the maximum $t$-value between C08 and C45 occurs at the offset of +20 rings and the $t$-value is then 5.6.

The site sequence is composed of the average of the corresponding widths, as illustrated with one width.
have taken place between AD 1512 and 1515 , which is much more precise than without this extra information.

Even if all the sapwood rings are missing on a sample, but none of the heartwood rings are, then an estimate of the felling-date range is possible by adding on the full compliment of say, 15 to 35 years to the date of the last heartwood ring (called the heartwood/sapwood boundary or transition ring and denoted H/S). Fortunately it is often easy for a trained dendrochronologist to identify this boundary on a timber. If a timber does not have its heartwood/sapwood boundary, then only a post quem date for felling is possible.
5. Estimating the Date of Construction. There is a considerable body of evidence collected by dendrochronologists over the years that oak timbers used in buildings were not seasoned in medieval or early modern times (English Heritage 1998 and Miles 1997, 50-55). Hence provided all the samples in a building have estimated felling-date ranges broadly in agreement with each other, so that they appear to have been felled as a group, then this should give an accurate estimate of the period when the structure was built, or soon after (Laxton et al 2001, figure 8 and pages 34-5 where 'associated groups of fellings' are discussed in detail). However. if there is any evidence of storing before use or if there is evidence the oak came from abroad (eg Baltic boards), then some allowance has to be made for this.
6. Master Chronological Sequences. Ulimately, to date a sequence of ring widths, or a site sequence, we need a master sequence of dated ring widths with which to cross-match it, a Master Chronology. To construct such a sequence we have to start with a sequence of widths whose dates are known and this means beginning with a sequence from an oak tree whose date of felling is known. In Fig 6 such a sequence is SHE-T, which came from a tree in Sherwood Forest which was blown down in a recent gale. After this other sequences which cross-match with it are added and gradually the sequence is 'pushed back in time' as far as the age of samples will allow. This process is illustrated in Fig 6 . We have a master chronological sequence of widths for Nottinghamshire and East Midlands oak for each year from AD 882 to 1981. It is described in great detail in Laxton and Litton (1988), but the components it contains are shown here in the form of a bar diagram. As can be seen, it is well replicated in that for each year in this period there are several sample sequences having widths for that year. The master is the average of these. This master can now be used to date oak from this area and from the surrounding areas where the climate is very similar to that in the East Midlands. The Laboratory has also constructed a master for Kent (Laxton and Litton 1989). The method the Laboratory uses to construct a master sequence, such as the East Midlands and Kent, is completely objective and uses the Litton-Zainodin grouping procedure (Laxton et al 1988). Other laboratories and individuals have constructed masters for other areas and have made them available. As well as these masters, local (dated) site chronologies can be used to date other buildings from nearby. The Laboratory has hundreds of these site sequences from many parts of England and Wales covering many short periods.
7. Ring-width Indices. Tree-ring dating can be done by cross-matching the ring widths themselves, as described above. However, it is advantageous to modify the widths first. Because different trees grow at different rates and because a young oak grows in a different way from an older oak, irrespective of the climate, the widths are first standardized before any matching between them is attempted. These standard widths are known as ring-width indices and were first used in dendrochronology by Baillie and Pilcher (1973). The exact form they take is explained in this paper and in the appendix of Laxton and Litton (1988) and is illustrated in the graphs in Fig 7. Here ring-widths are plotted vertically, one for each year of growth. In the upper sequence of (a), the generally large early growth after 1810 is very apparent as is the smaller later growth from about 1900 onwards when the tree is maturing. A similar phenomena can be observed in the lower sequence of (a) starting in 1835 . In both the widths are also changing rapidly from year to year. The peaks are the wide rings and the troughs are the narrow rings corresponding to good and poor growing seasons, respectively. The two corresponding sequence of Baillie-Pilcher indices are plotted in (b) where the differences in the immature and mature growths have been removed and only the rapidly changing peaks and troughs remain, that are associated with the common climatic signal. This makes cross-matching easier.


Fig. 6 Bar diagram showing the relative positions and dates of the first rings of the component site sequences in the East Midlands Master Dendrochronological Sequence, EM08/87
(a)

(b)


Fig 7. (a) The raw ring-widths of two samples, THO-A01 and THO-B05, whose felling dates are known. Here the ring widths are plotted vertically, one for each year, so that peaks represent wide rings and troughs narrow ones. Notice the growth-trends in each; on average the earlier rings of the young tree are wider than the later ones of the older tree in both sequences.

Fig 7. (b) The Baillie-Pilcher indices of the above widths. The growth-trends have been removed completely.

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